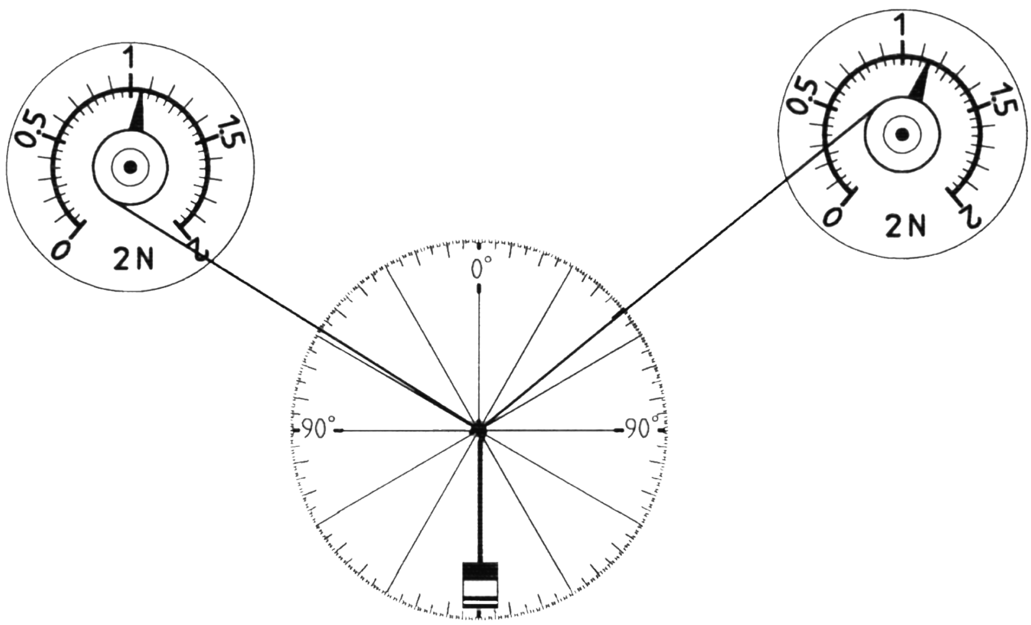
Demonstration Experiments

Demo advanced

Magnet Board Mechanics 1



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DEMO advanced
Demonstration Experiments
Magnet Board Mechanics 1

Order No. 01152-02

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DEMO advanced Demonstration Experiments
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1st edition

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	Mass and weight	MT 1.1

With the aid of a spring dynamometer, demonstrate that the weight of a body is proportional to its mass.

Equipment

Demonstration board for physics	02150.00	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver	02205.02	2
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2

Set-up and procedure

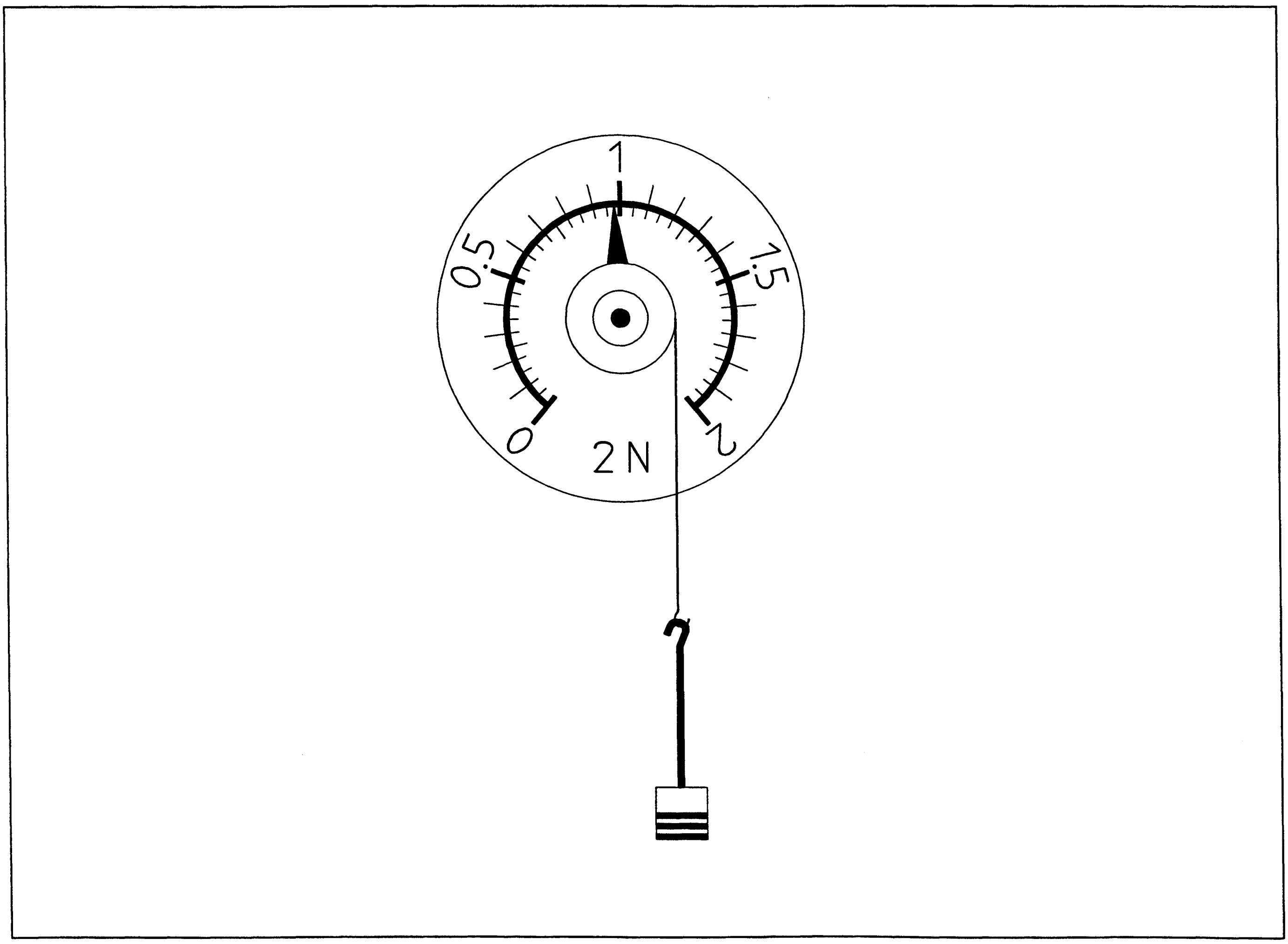
— Set-up the experiment according to Fig. 1.
— Set the pointer of the dynamometer to zero.
— Put 4 slotted weights (each 10 g; alternating colours) onto the weight holder and hang it on the dynamometer.
— Read the weight F_G and record it in Table 1.
— Successively place the 50 g slotted weights (alternating colours) onto the weight holder; note the respective weight and enter it in Table 1.

Results

Table 1

m/g	F_G/N	m/kg	$\frac{F/m}{N/kg}$
50	0.49	0.05	9.8
100	0.97	0.10	9.7
150	1.45	0.15	9.7
200	1.95	0.20	9.8

Fig. 1



Evaluation

The graph of the measured values (Fig. 2) is a straight line through the origin of the F_G-m co-ordinate system.

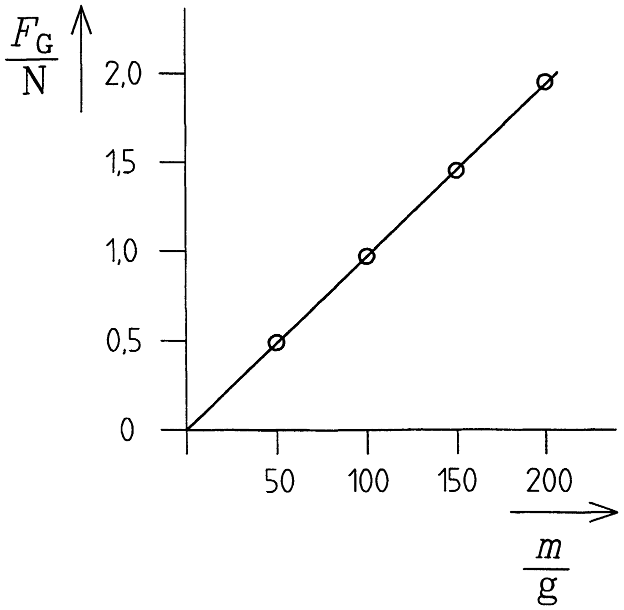
Therefore, it follows that there is a proportional correlation between the weight F_G and the mass m in the experiment:

$$F_G \sim m.$$

This correlation is equivalent to F_G/m = constant. This relationship can be subsequently confirmed by forming quotients (cf. Table 1, 4th column): If the mass of the test object in kg is substituted in the quotient formation, as in Table 1, one obtains the following as an approximate mean:

$$F_G/m = 9.8 \text{ N/kg}.$$

Fig. 2



A body with a mass of 1 kg experiences a weight of 9.8 N. Or: 1 N is the weight, which a body with a mass of 102 g experiences.

Remarks

The value of 9.8 N/kg corresponds to the acceleration of gravity g. If one writes the experimental result in the form

$$F_G = m \cdot g,$$

one obtains a special form of Newton's law $F = m \cdot a$. From this one can conclude the following:
The weight $\vec{F}_G$ imparts the acceleration $\vec{g}$ (the acceleration of gravity) to a free-falling body. The acceleration of gravity is a function of the location. For Central Europe the following is valid:

$$g = 9.81 \text{ m/s}^2.$$

These considerations can be adopted to the treatment of free fall in kinematics and in connection with the treatment of Newton's law. In this context, the newton can also be introduced as a unit:

$$1 \text{ N} = 1 \text{ kg/s}^2.$$

The first treatment of the correlation between weight and mass should be restricted to the above-mentioned, descriptive version of the results, i.e. that 1 N is the weight that a body with a mass of 102 g experiences (e.g. a 100 g chocolate bar with wrapper.

Extension of a rubber band and helical spring

Through progressive extension of a rubber band and a helical spring, demonstrate the difference between plastic and elastic deformation.

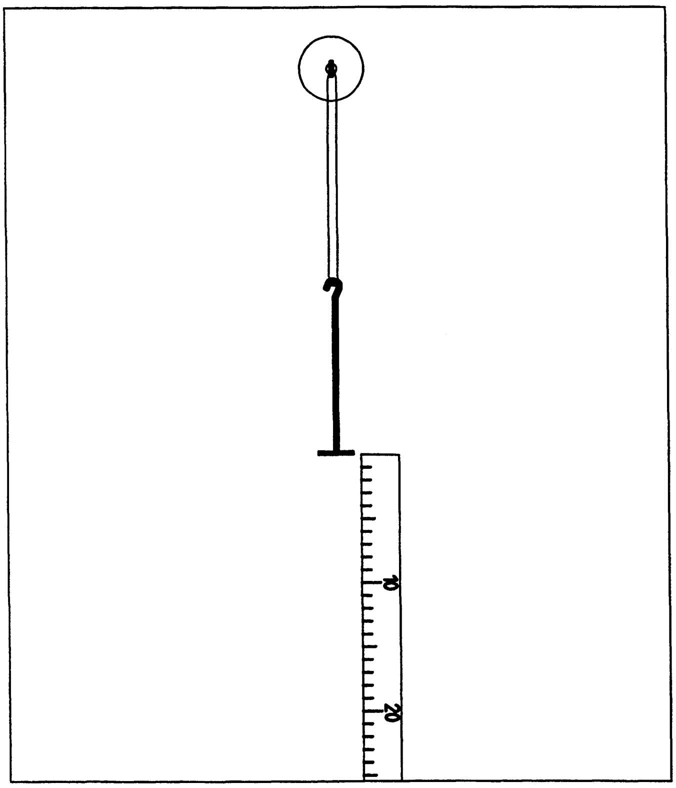
Equipment

Demonstration board for physics	02150.00	1
Hook on fixing magnet	02151.03	1
Scale for demonstration board	02153.00	1
Helical spring, 20 N/m	02222.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver	02205.02	3
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Rubber bands, 1 piece out of	03920.00	(1)

Set-up and procedure 1

- Position the hook on the fixing magnet near the upper edge of the demonstration board and hang the rubber band on the hook.
- Preload the rubber band by hanging the weight holder on it.
- Position the scale on the demonstration board in such a manner that the bottom of the weight holder is at the same height as the zero mark of the scale (Fig. 1).

Fig. 1



- Place all five 10-g slotted weights onto the weight holder; measure the resulting extension *s* and record this measured value in Table 1.
- Place the three 50-g slotted weights successively onto the weight holder, and measure the respective extension in each case, and record the values in Table 1. (Note: It is advisable to wait a few minutes before reading the last measured value because the rubber band continues to stretch slightly for some time) at larger loads.)
- Reduce the loading of the rubber band progressively (by 50 g in each case, measure the respective extension, and record the values in Table 1.

Set-up and procedure 2

- Attach the 20 N/m helical spring to the hook in place of the rubber band.
- Determine the extension s with progressive loading and unloading of the helical spring in the same manner as in experiment 1, and record the measured values in Table 2.

Results 1

Table 1 (100 g $\hat{=}$ 0.98 N)

m / g	*F* / N	Extension during Loading *s* / cm	Extension during Unloading *s* / cm
0	0	0	0.3
50	0.49	1.1	1.9
100	0.98	3.0	4.3
150	1.47	6.1	7.1
200	1.96	9.3	9.3

Results 2

Table 2 (100 g $\hat{=}$ 0.98 N)

m / g	*F* / N	Extension during Loading *s* / cm	Extension during Unloading *s* / cm
0	0	0	0
50	0.49	2.6	2.6
100	0.98	5.2	5.2
150	1.47	7.6	7.7
200	1.96	10.2	10.2

Evaluation 1

Table 1 and the graph of the measured values in Fig. 2 demonstrate the following:

— The extension *s* of rubber is not directly proportional to the force which causes the extension.

— After cessation of the action of force, rubber does not completely return to its original dimensions. If a force acts on a body made of rubber, a predominantly reversible change in shape occurs (in which, however, there is no proportionality between the deformation and the force), which is accompanied by an irreversible dimensional change (plastic deformation).

Evaluation 2

Table 2 and the graph of the measured values in Fig. 3 establish the following:

— The extension *s* of steel is proportional to the force which causes the extension.

— After cessation of the action of the force, steel returns completely to its original dimensions.

If a force acts on a body made of steel, an entirely reversible dimensional change occurs (elastic deformation).

Remarks

The graphs in Figures 2 and 3 represent the *s-F* characteristic curves of the bodies which were deformed (in this case stretched) by forces in Experiments 1 and 2.

If the extension is accompanied by plastic deformation, the characteristic curves which are obtained for increasing (loading) and decreasing force (unloading) differ from each other. In elastic deformation, i.e. for which $s \sim F$ is valid, the characteristic curves coincide.

The measured values determined in Experiment 1 can only be considered as sample values since they are not only a function of the rubber band employed (age, dimensions, material), but also a function of how rapidly the reading of the indicated values is performed subsequent to the respective force changes. Above all, when using large forces the rubber continues to stretch somewhat for a time. Therefore, it is advisable to wait a short time (circa 1 min) before noting the measured value for the maximum extension and to begin with the load alleviation of the rubber band immediately after reading the value.

For the spring, the proportionality between *F* and *s* is only valid up to a characteristic limiting force. The region of proportionality is generally smaller than the region of elasticity. If the region of elasticity is exceeded in the deformation of elastic bodies, plastic deformation also occurs in this case.

In these experiments, weight was chosen as the deforming force and it was assumed that the students know the relationship 100 g $\hat{=}$ 0.98 N. One can however also proceed from the relation 1 N $\hat{=}$ 100 g and work with more simple numerical values in the tables and figures: this does not change the fundamental results of the experiments in any way.

Fig. 2

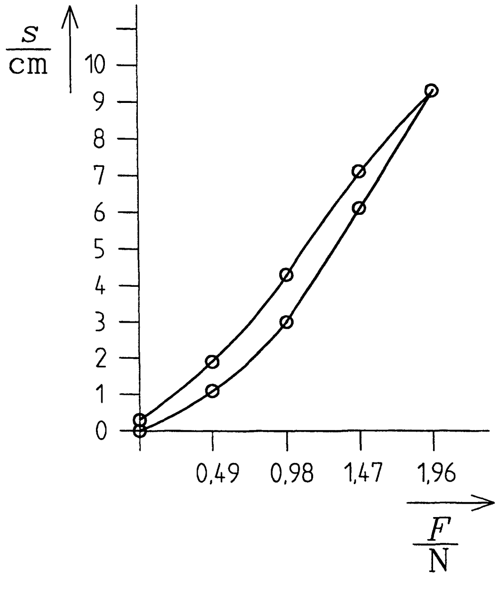
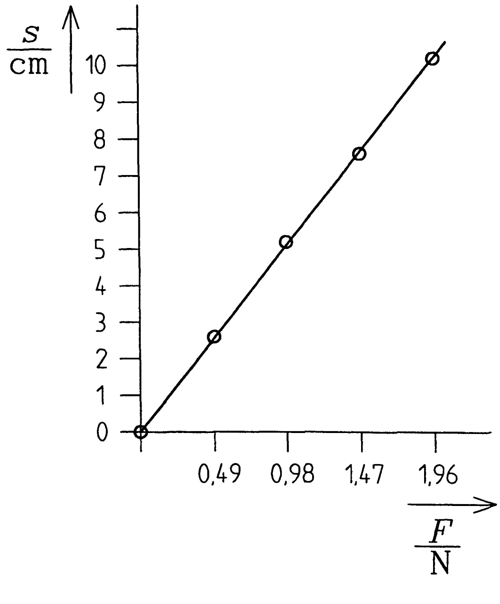


Fig. 3



Show that Hooke's law is valid in the loading of a helical spring and that the extension of the springs is a function of the acting force and of the hardness of the springs.

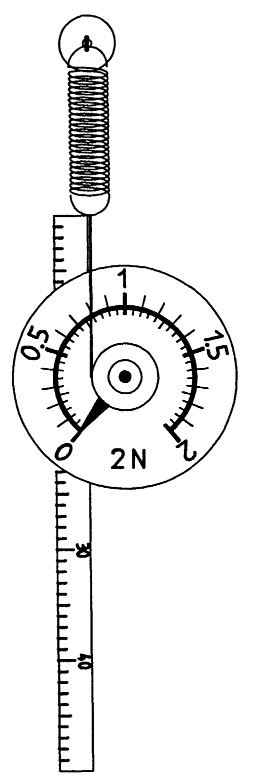
Equipment

Demonstration board for physics	02150.00	1
Hook on fixing magnet	02151.03	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Helical spring, 3 N/m	02220.00	1
Helical spring, 20 N/m	02222.00	1

Set-up

— Position the hook on the fixing magnet near the upper edge of the demonstration board and hang the soft helical spring with 3 N/m onto it.
— Place the dynamometer directly below the helical spring; hook its traction cord onto the helical spring; turn the dynamometer in such a manner that the line is as short as possible and that the spring is slightly preloaded.

Fig. 1



— Position the scale on the demonstration board such that the lower end of the spring is at the same height as the zero mark of the scale.
— Set the pointer of the dynamometer to zero, and secure the scale (Fig. 1).

Procedure

— Move the dynamometer downward until it indicates 0.2 N; measure the resulting extension s and record it in Table 1.
— Increase the tractive force in further steps of 0.2 N each. (When the dynamometer has reached the lower edge of the demonstration board, turn the dynamometer slightly, if necessary, and thus wind up the cord until a value of F = 0.8 N is reached.) Measure the respective extension and note it in Table 1.
— In a corresponding manner, perform the experiment with the hard helical spring with 20 N/m (until F= 1.2 N), and record the measured values in Table 2.

Results

Table 1: Soft spring.

F / N	s / cm	s / m	$\frac{F/s}{\text{N/m}}$
0	0	0	–
0.2	6.6	0.066	3.0
0.4	12.8	0.128	3.1
0.6	19.2	0.192	3.1
0.8	25.5	0.255	3.1

Table 2: Hard spring.

F / N	s / cm	s / m	$\frac{F/s}{\text{N/m}}$
0	0	0	–
0.2	1.0	0.010	20.0
0.4	2.0	0.020	20.0
0.6	3.0	0.030	20.0
0.8	4.1	0.041	19.5
1.0	5.0	0.050	20.0
1.2	6.0	0.060	20.0

Evaluation

The graph of the measured values (Fig. 2) is a straight line in each case. Therefore, the following is true:

$$F \sim s.$$

This is equivalent to F/s = constant, which can be confirmed by forming quotients (cf. Table 1 and 2, last column). The deformation (extension) of the helical spring is proportional to the deforming force. The two results differ in that the same force results in an extension that is more than 6 times as large for the first spring investigated as for the second one. The springs are said to differ in their hardness, and this is expressed by the spring constant:

$$D = F/s.$$

The measurements determine a spring constant of approximately 3.1 N/m for the soft spring which was first investigated; for the hard, second spring one of approximately 20 N/m (cf. list of materials).
The relationship $F \sim s$ is called Hooke's law. With $F/s = D$ one obtains Hooke's law in the following form:

$$F = D \cdot s.$$

Remarks

While making the measurements, one must take particular care to ensure that the parallax errors when reading the values for s as well when on setting the pointer on the dynamometer are as small as possible.
Elastic bodies are characterised by the fact that they regain their original dimensions after deformation by a force when no further force acts on them. Their deformation is thus reversible, but only if the force does not exceed a specific value. The liner relationship $F \sim s$ or $F = D \cdot s$, which was discovered by Robert Hooke, is a special case of elastic deformation since there are also bodies made of materials which also show a non-liner force-deformation behaviour in their elastic region.
$F = D \cdot s$ is valid in such cases only for the so-called proportionality region.
In this experiment the helical spring was stretched with a dynamometer in order to be able to determine the relationship between displacement and force directly. It is also possible to stretch the helical springs by hanging "mass pieces" (weights) on them (Fig. 3). In this case, the respective weight must be calculated for the evaluation.

Fig. 2

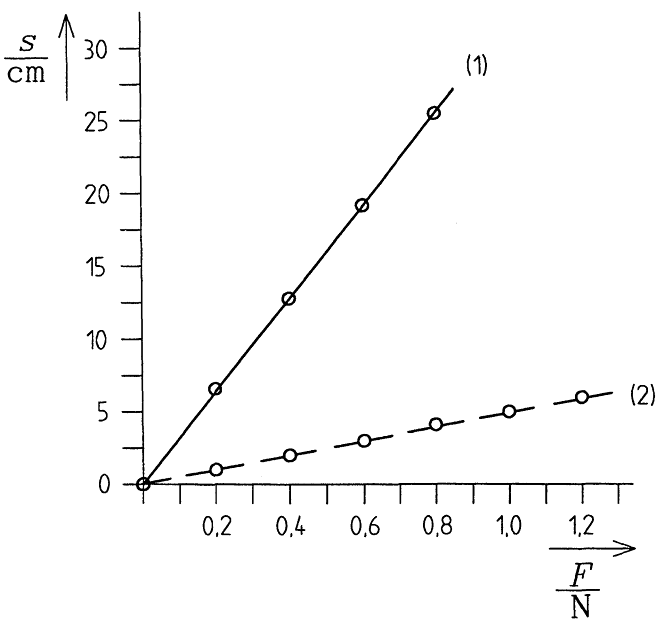
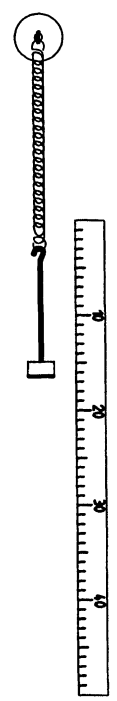


Fig. 3



Make and calibrate a spring dynamometer using a helical spring. Subsequently, determine the weight in N of a test piece with the model.

Equipment

Demonstration board for physics	02150.00	1
Hook on fixing magnet	02151.03	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 2 out of	02204.00	1
Helical spring, 20 N/m	02222.00	1
Fish line, 1 m of	02090.00	1
Test piece, e.g. lever (03960.00)		
White board pen, water soluble		

Set-up and procedure

— Place the hook on fixing magnet near the upper edge of the demonstration board and hang the helical spring onto it

— Position the dynamometer under it, set the pointer to zero, and secure the scale.

— Mark the position of the lower end of the helical spring with a pointer for the demonstration board (Fig. 1)

— Move the dynamometer downwards until it indicates a tractive force of 2 N.

— Mark the present position of the helical spring's lower end with the second pointer.

— Remove the dynamometer.

— Draw a scale for the dynamometer model: make a 0 index mark at the upper pointer, a 2-N index mark at the lower pointer. Determine the distance between the two marks with the scale for the demonstration board. Divide this distance (measuring range!) into 4 (or 20 – depending on the desired accuracy) equal sections.

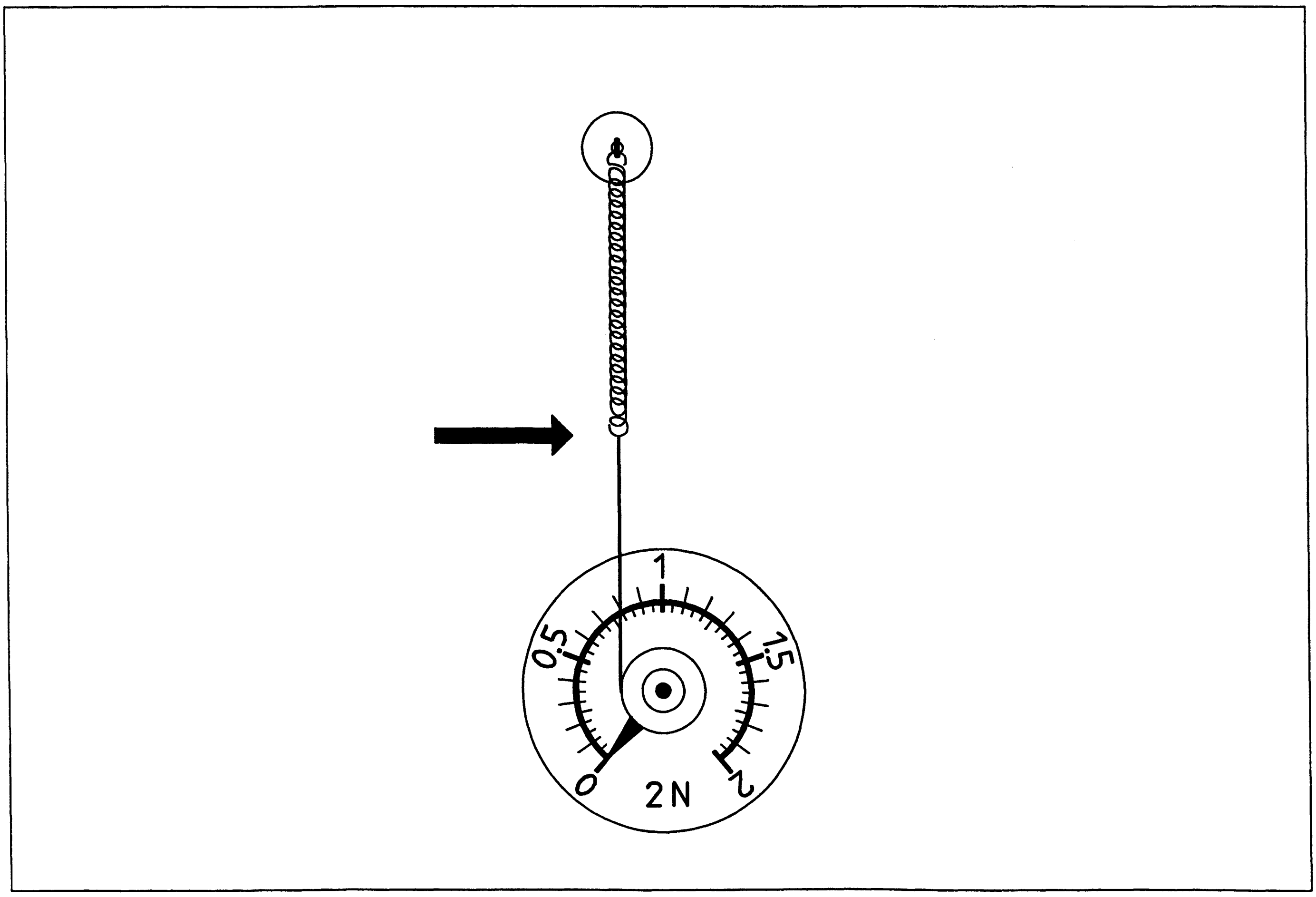
— Remove the pointers for the demonstration board and hang the test piece, e.g. the lever) on the helical spring

— Determine the weight of the test piece.

Results

F_G of the test piece (lever): 1.3 N

Fig. 1



Evaluation
Forces can be measured with a helical spring. To do this, the springs must be calibrated, i.e. (in this case) to determine which extension of the spring corresponds to which force. The results are recorded as a scale which is appropriately gradated.

Remarks
The gradations of the dynamometer's scale are only linear and thus particularly simple if the proportionality region for the object used for force measurement is not exceeded (and Hook's Law is valid) – which is the case in this experiment.
At this location one can also mention the measuring principle of the dynamometer, which contains a spiral spring instead of a helical spring.
The term calibration was used in this experiment because it only deals with the determination of the correlation between the initial and final parameters of a measuring device. The term standardise would not be accurate because this is understood to mean an official certification of measuring devices or measuring means.

Bending a leaf spring

Investigate the bending behaviour of a leaf spring under the conditions that the point of application and the direction of the force remain the same.
In addition, demonstrate that the action of the force is greatest when the force acts perpendicularly to the leaf spring.

Equipment

Demonstration board for physics	02150.00	1
Clamp on fixing magnet	02151.01	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 2 out of	02204.00	1
Protractor disk, magnet held	08270.09	1
Leaf spring, 300 mm x 15 mm	02228.00	1
White board pen, water soluble		

Set-up
— Place the clamp on fixing magnet onto the demonstration board and clamp the leaf spring into a horizontal position with it.
— Positon the two pointers in such a manner that their lateral edges are at the same height as the horizontally positioned leaf spring (Fig. 1).
— Place and adjust the dynamometer in such a manner that its traction cord is vertical.

Procedure 1
— Move the dynamometer vertically downwards until it indicates a force of 0.1 N. With the white board pen mark the point on the board above which the end of the leaf spring is located.
— Move the dynamometer further downwards and in each case shift it slightly to the side (so that the traction cord always remains vertical), and proceed in 0.1-N steps as above.
— Remove the dynamometer and with the aid of the scale determine the (vertical) distances *s* between the points marked with the white board pen and the lower edge of the scale. Record the values for *s* in Table 1.

Procedure 2
— Position the dynamometer on the lower part of the demonstration board is such a manner that the traction cord remains vertical, and a force of 0.8 N is indicated (Fig. 2).
— Place the protractor disk such that its centre is directly behind the end of the leaf spring.

Fig. 1

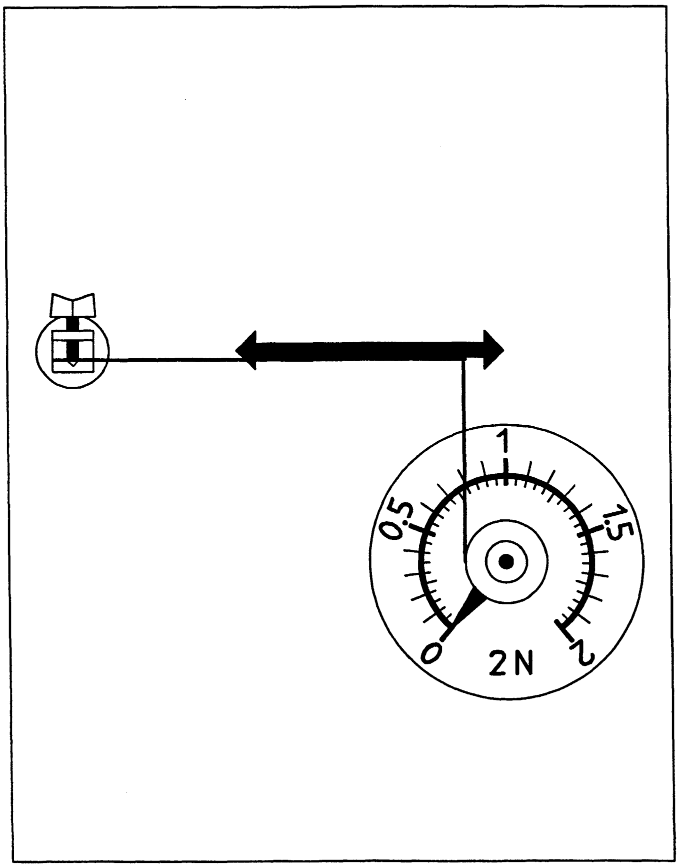
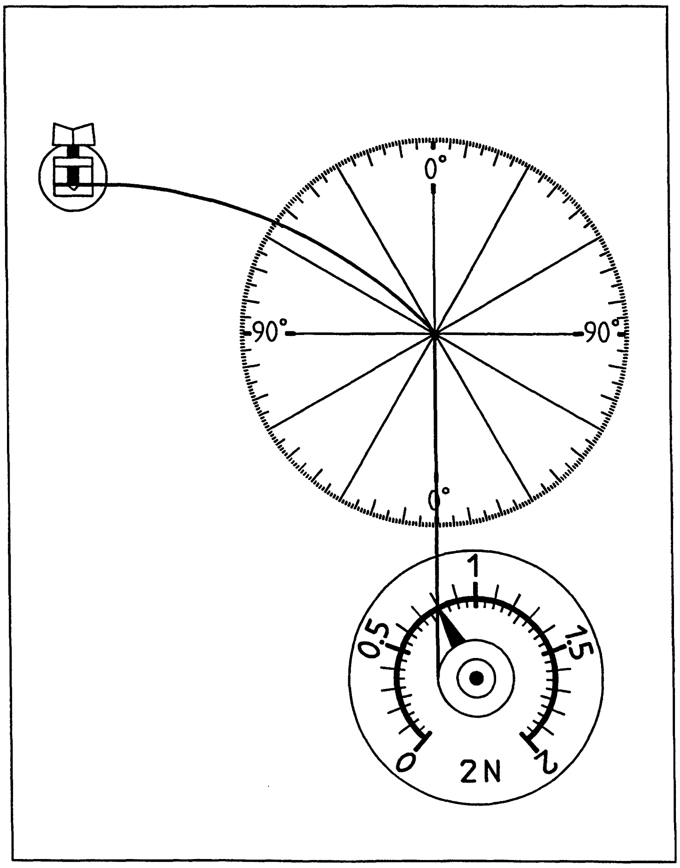


Fig. 2



— Now shift the dynamometer progressively in the horizontal plane (to the right and left; and, if necessary, move it slightly vertically in such a manner that the traction cord forms an angle of approximately 30°, 45°, ... with the vertical line and the end of the leaf spring always remains centred on the protractor disk (*s* = constant). Measure the required force in each case and note the values for the horizontal traction cord and the traction cord perpendicular to the bent spring

Results 1

Table 1

F / N	*s* / cm
0.1	2.3
0.2	4.5
0.3	6.3
0.4	7.9
0.5	9.4
0.6	10.9
0.7	12.1
0.8	13.0

Results 2

Table 2

Traction cord	*F* / N
Perpendicularly downwards	0.8
Horizontally	1.2
Perpendicular to leaf spring	0.65

Evaluation

The graph of the measured values in Table 1 results in a curve which is nearly linear in its lower segment, but then exhibits an increasingly strong curvature in its upper region (Fig. 3)

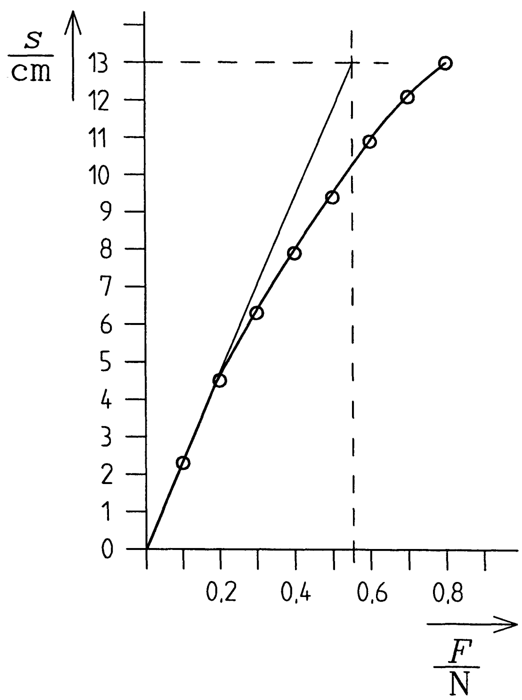
The elastic deformation is also a function of the force direction. The force required for a desired displacement is the smallest when it acts perpendicular to the spring. In other tractive directions, only the components which act perpendicular to the spring are effective.

This explains why the curve in Fig. 3 deviates increasingly strongly from a straight line. If one considers the measured value recorded in Table 2 for the force acting perpendicular to the spring in Fig. 3, this point lies on the straight line on which the points indicating the values for small displacements are located.

Remarks

The experimental set-up can additionally serve to demonstrate the dependence of the force's action on the point of application of the force. One allows a constant force to act on different locations on the leaf spring. In this case, the experimental set-up as a whole can be used to make the students aware that forces are characterised by quantity, direction and point of application.

Fig. 3



Force and counterforce

Show that a force which acts on a resting body always induces an equally large counterforce.

Equipment

Demonstration board for physics	02150.00	1
Hook on fixing magnet	02151.03	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Pointers for demonstration board, 2 out of	02204.00	1
White board pen, water soluble		

Set-up and procedure

— Place the hook on fixing magnet and a dynamometer onto the demonstration board.

— Attach the traction cord of the dynamometer to the hook. Adjust the dynamometer and then shift it to the right until it indicates 2 N (Fig. 1, upper part).

— After the question as to why the dynamometer indicates 2 N (cf. Result (1)) has been answered, replace the hook on fixing magnet with the second dynamometer. Hook the two traction cords together, and adjust the dynamometers (Fig. 1, lower part)

— Shift the left dynamometer to the left until the right dynamometer against indicates 2 N.

— Read and record the force indicated by the left dynamometer (Result (1)).

— After determination of the results, place the pointers for the demonstration board (one red and one blue one) onto the board. Label them with the white board pen (cf. Fig. 1, lower part)

Results

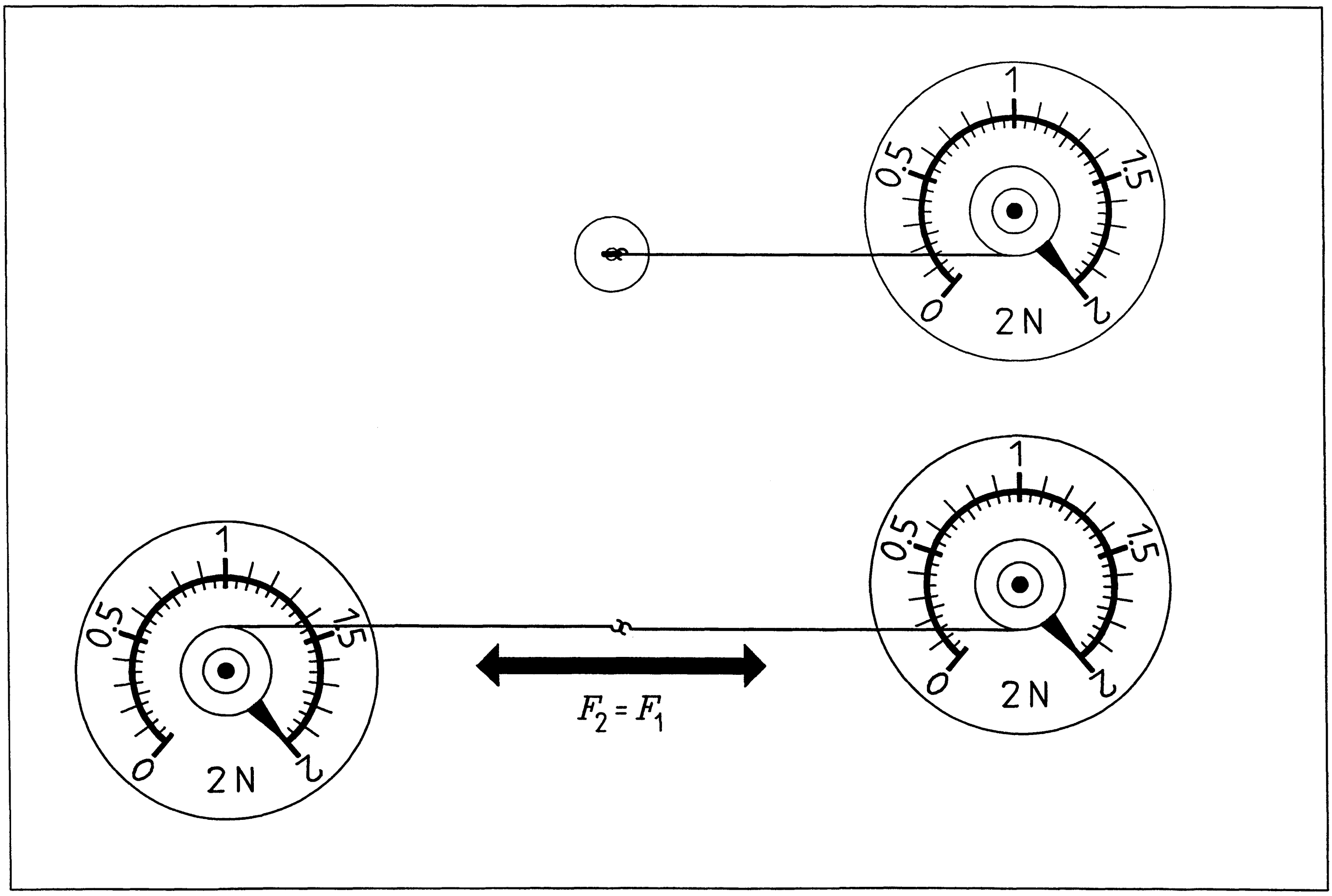
(1) The dynamometer pulls with 2 N on the hook. The hook remains at rest. Obviously it is able to withstand the 2 N force, i.e., it pulls – also with 2 N – on the dynamometer.

(2) The left dynamometer indicates a force equal to that on the right one, i.e. 2 N.

Evaluation

If a force acts on a body, it always induces an equally large force directed in the opposite direction.

Fig. 1



This law is named after its discoverer as Newton's Third Law and presented in the form

Force = counterforce

or action = reaction.

It is also known as the law of reciprocal action.

Remark

If the students are already familiar with vectorial notation for the designation of forces or if one desires to introduce it at this point in the course, it is advisable to present it complementarily on the demonstration board below $F_1 = F_2$, i.e. $\vec{F}_1 = -\vec{F}_2$.

Composition of forces having the same line of application

Demonstrate how forces which have the same line of application and the same or opposite direction are composed.

Equipment

Demonstration board for physics	02150.00	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 4 out of	02204.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver	02205.02	2
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
White board pen, water soluble		

Set-up and procedure 1

- Place a dynamometer in the centre of the demonstration board and adjust it.
- Hang the weight holder with slotted weights (4 x 10 g, 3 x 50 g) on the dynamometer. Read the indicated force (F_G) and record it (1).
- Place the second dynamometer above the first one in such a manner that its tension cord (nearly) touches the cord reel of the first dynamometer. Adjust the dynamometer and hook its traction cord onto the weight holder.
- Move the second dynamometer upwards until it indicates 0.5 N (F_2) (Fig. 1). Read the force (F_1) which is now indicated by the first dynamometer and record the values for the two forces (2).
- With the white board pen and with the aid of the scale, draw the force vectors (arrows) for $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_G$ (lengths proportional to their magnitudes).
- Remove the devices from the demonstration board and determine F_R on the demonstration board (cf. Fig. 3).

Set-up and procedure 2

- Place a dynamometer (4 N measuring range) near the upper edge of the demonstration board and adjust it.
- Hang the weight holder with slotted weights (4 x 10 g, 3 x 50 g) on the dynamometer. Read the indicated force (F_G) and record it (1).
- Place the second dynamometer (2 N measuring range) as illustrated in Fig. 2. Hook the traction cord onto the weight holder, and adjust the dynamometer
- Move the 2 N dynamometer downwards until it indicates a force of 1.5 N (F_2).
- Read the force indicated by the upper dynamometer (4 N) (F_1)
- Record F_1 and F_2 (2).

Fig. 1

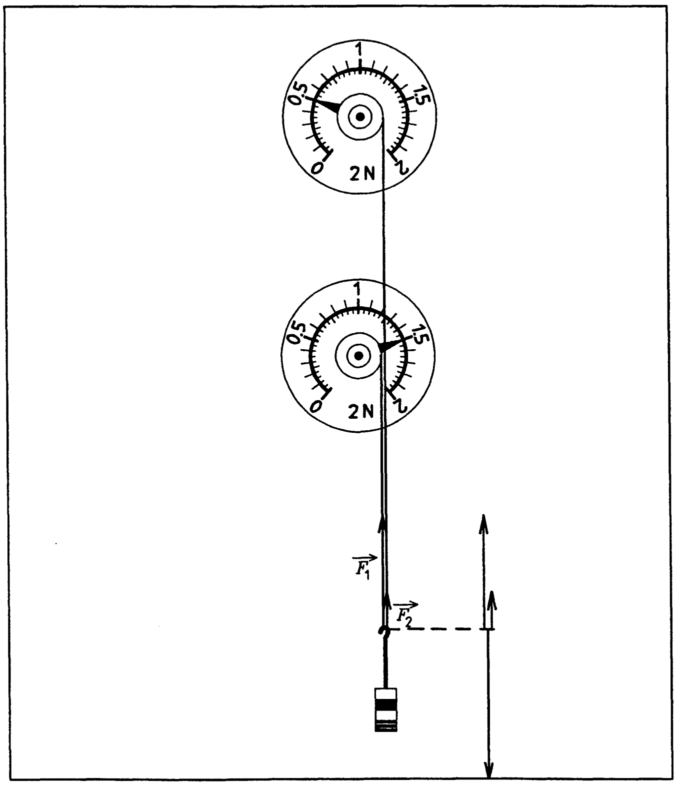
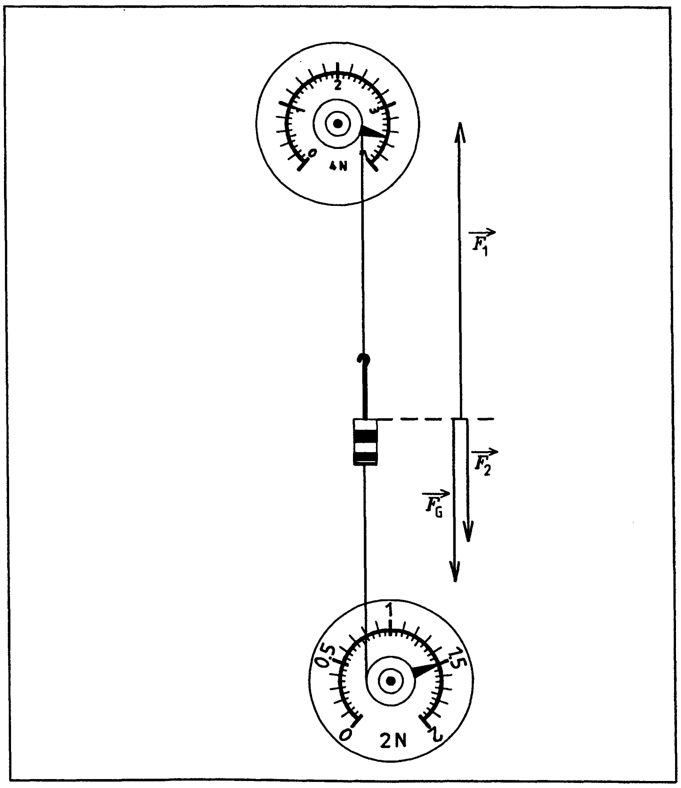


Fig. 2



— Draw in the force vectors (arrows) and proceed as in Part 1 of this experiment(cf. Fig. 4).

Results 1

(1) $F_G = 2N$
(2) $F_2 = 0.5$ N; $F_1 = 1.5$ N

Results 2

(1) $F_G = 2N$
(2) $F_2 = 1.5$ N; $F_1 = 3.5$ N

Evaluation 1

A comparison of forces shows that an upward force resulting from the addition of $\vec{F}_1$ and $\vec{F}_2$ balances the weight $\vec{F}_G$. This force is termed the resultant force $\vec{F}_R$. Therefore, the following is true:

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2,$$

and in this special case F_R balances F_G;

$$F_R = F_1 + F_2 = F_G.$$

Forces on the same line of action which are oriented in the same direction can be combined by adding their values.

Evaluation 2

The comparison of the forces in this case results in the following:

$$F_R = F_1 - F_2 = F_G.$$

Forces on the same line of action which are oriented in opposite directions can be combined by subtracting one value from the other.

Remarks

Forces are vectorial quantities. If two forces act simultaneously on one point, then the following is true:

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2.$$

The resultant force is determined graphically by superimposing the force vectors (arrows) on each other and in the experimentally determined cases one calculates their magnitude by adding or subtracting the magnitude of the components $\vec{F}_1$ and $\vec{F}_2$.

As a result of the relationship 102 $\hat{=}$ 1 N, the given measured values have an inaccuracy of 2%. This degree of accuracy is however sufficiently exact for this experiment.

Fig. 3

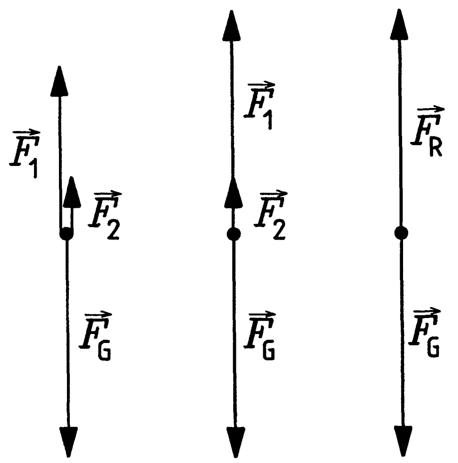
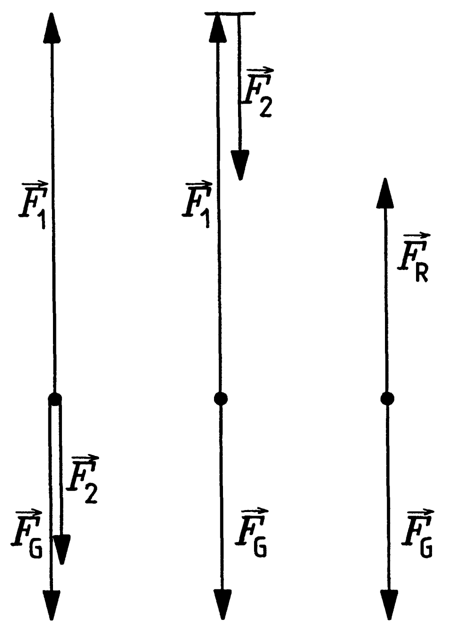


Fig. 4



Investigate how the resultant force of two forces can be determined if their lines of action are not parallel.

Equipment

Demonstration board for physics	02150.00	1
Hook on fixing magnet	02151.03	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Helical spring, 20 N/m	02222.00	1
Protractor disk, magnet held	08270.09	1
White board pen, water soluble		

Set-up and procedure

— Place the hook on fixing magnet near the upper edge of the demonstration board and hang the helical spring on it.

— Place the two dynamometers below the spring in such a manner that their traction cords, whose free ends are hooked to the lower end of the helical spring, sag slightly.

— Adjust both dynamometers and then move them so that the helical spring is stretched, e.g. by approximately 7 cm.

— Place the protractor disk onto the demonstration board in such a manner that its centre is exactly behind the lower end of the helical spring (Fig. 1).

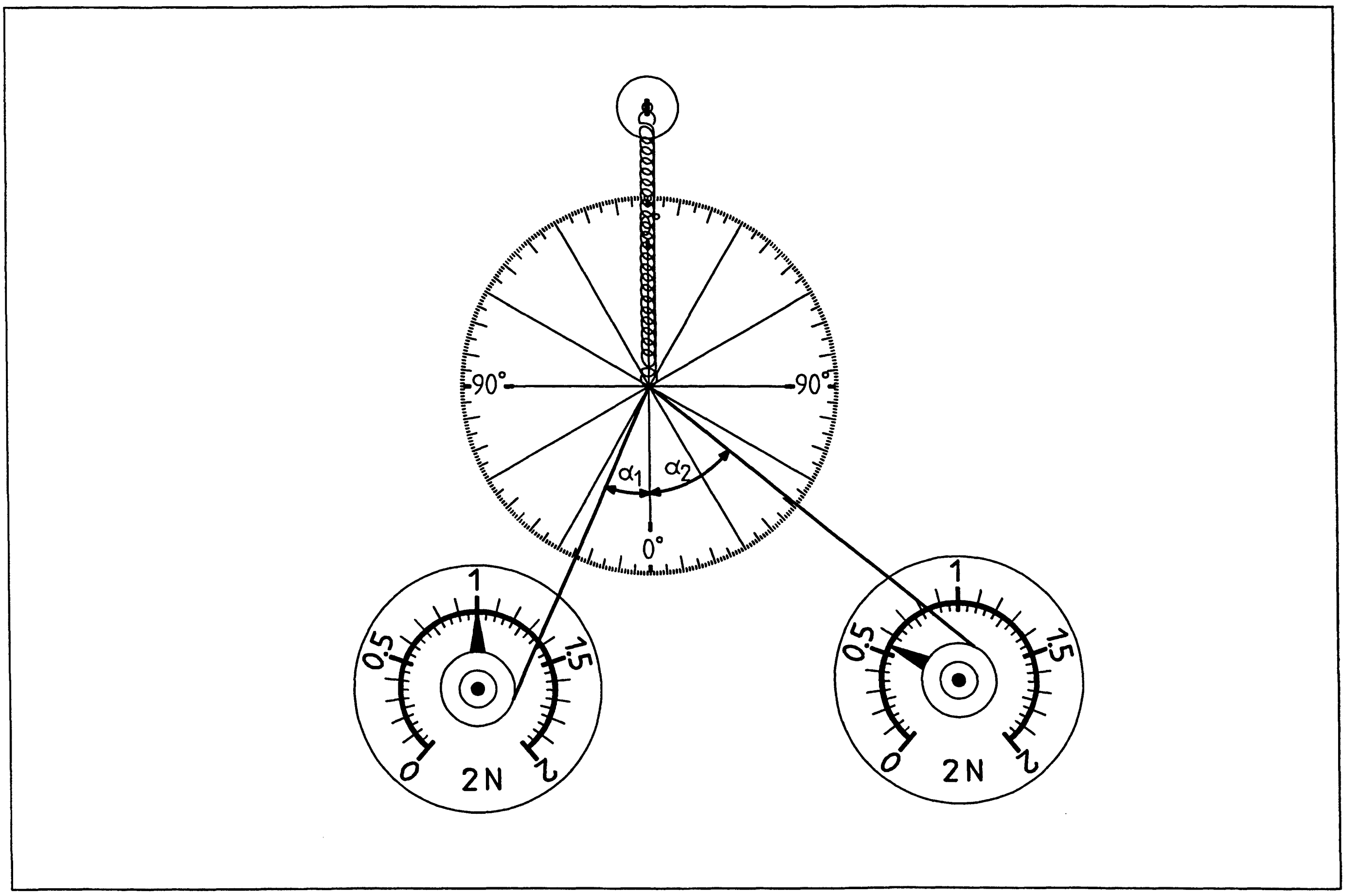
— Read values for the forces F_1 and F_2 indicated by the dynamometers and the angles α_1 and α_2, which are between their lines of action and the perpendicular to the horizontal line of the protractor disk.

— Note the results in Table 1.

— Change the positions of the dynamometers, but not that of the protractor disk. Record the respective magnitudes of $\vec{F_1}$ and $\vec{F_2}$ as well as the corresponding α_1 and α_2 (including the case where $\alpha_1 + \alpha_2 = 90°$. While doing so, ensure that the lower end of the helical spring (the point of application of the forces) is located above the centre of the protractor disk. Record the measured values in Table 1.

— Subsequent to the last setting, remove one dynamometer and with the other one measure the force F_R which is required to stretch the helical spring to the centre of the protractor disk. Record the value for F_R.

Fig. 1



— For a second series of trials preset another extension of the helical spring, e.g., approximately 10 cm. Select different angles for $\vec{F}_1$ and $\vec{F}_2$, and record the values in Table 2. Finally, determine F_R again.
— For the graphical evaluation remove both dynamometers and construct the force parallelogram with the white board pen for one on the demonstration board of the investigated cases using the protractor disk and the scale (Fig. 2).

Results

Table 1 (Sample measurement)

$\frac{F_1}{N}$	$\frac{F_2}{N}$	$\frac{\alpha_1}{1°}$	$\frac{\alpha_2}{1°}$	$\frac{F_R}{N}$	$\frac{F_1+F_2}{N}$	$\frac{\alpha_1+\alpha_2}{1°}$
1.10	1.33	67	50	1.27	2.43	117
1.10	0.64	30	60	1.27	1.74	90
1.02	0.54	24	51	1.27	1.56	75
0.92	0.52	20	39	1.27	1.44	59

Table 2

$\frac{F_1}{N}$	$\frac{F_2}{N}$	$\frac{\alpha_1}{1°}$	$\frac{\alpha_2}{1°}$	$\frac{F_R}{N}$	$\frac{F_1+F_2}{N}$	$\frac{\alpha_1+\alpha_2}{1°}$
1.46	0.77	24	51	1.81	2.23	75
1.57	0.91	30	60	1.81	2.48	90

Evaluation

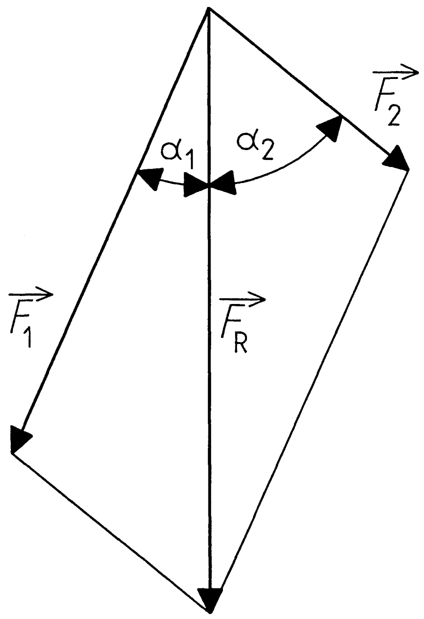
It can be seen from Tables 1 and 2 that the sum of the magnitudes of $\vec{F}_1$ and $\vec{F}_2$ are always larger than $\vec{F}_R$ and that the larger the angle enclosed by the angles $(\alpha_1 + \alpha_2)$, the larger their sum is.
In any case, $\vec{F}_1$ and $\vec{F}_2$ result in the same action as the force $\vec{F}$; for this reason $\vec{F}$ is termed the resultant force $\vec{F}_R$; $\vec{F}_1$ and $\vec{F}_2$ are termed its components.
$\vec{F}_R$ can be determined as the diagonal of a force parallelogram whose sides are formed by the components drawn to the same scale.

Two forces, whose lines of action intersect, i.e. which have a common point of application can be replaced by a single force. This can be determined by construction or calculation.

Remarks

It is advisable to have the students simultaneously construct the same force parallelogram in their notebooks while the teacher is drawing it on the demonstration board.
The special case in which $\alpha_1 + \alpha_2 = 90°$ was selected in order that the students could check their results with a sample calculation even without knowledge of trigonometry.
An additional task could be a graphical check of the remaining measurements.
Recording an exact series of measurements is not absolutely necessary. One can also restrict the experiment to a single measurement of F_1, F_2, α_1 and α_2, and determine the force parallelogram by quadrupling the values. In this case, one should however demonstrate qualitatively that the components enclose arbitrary angles and as a result can have differing values.

Fig. 2



Resolution of a force into two non-parallel forces

Demonstrate that one force can be resolved into two forces whose lines of action intersect.

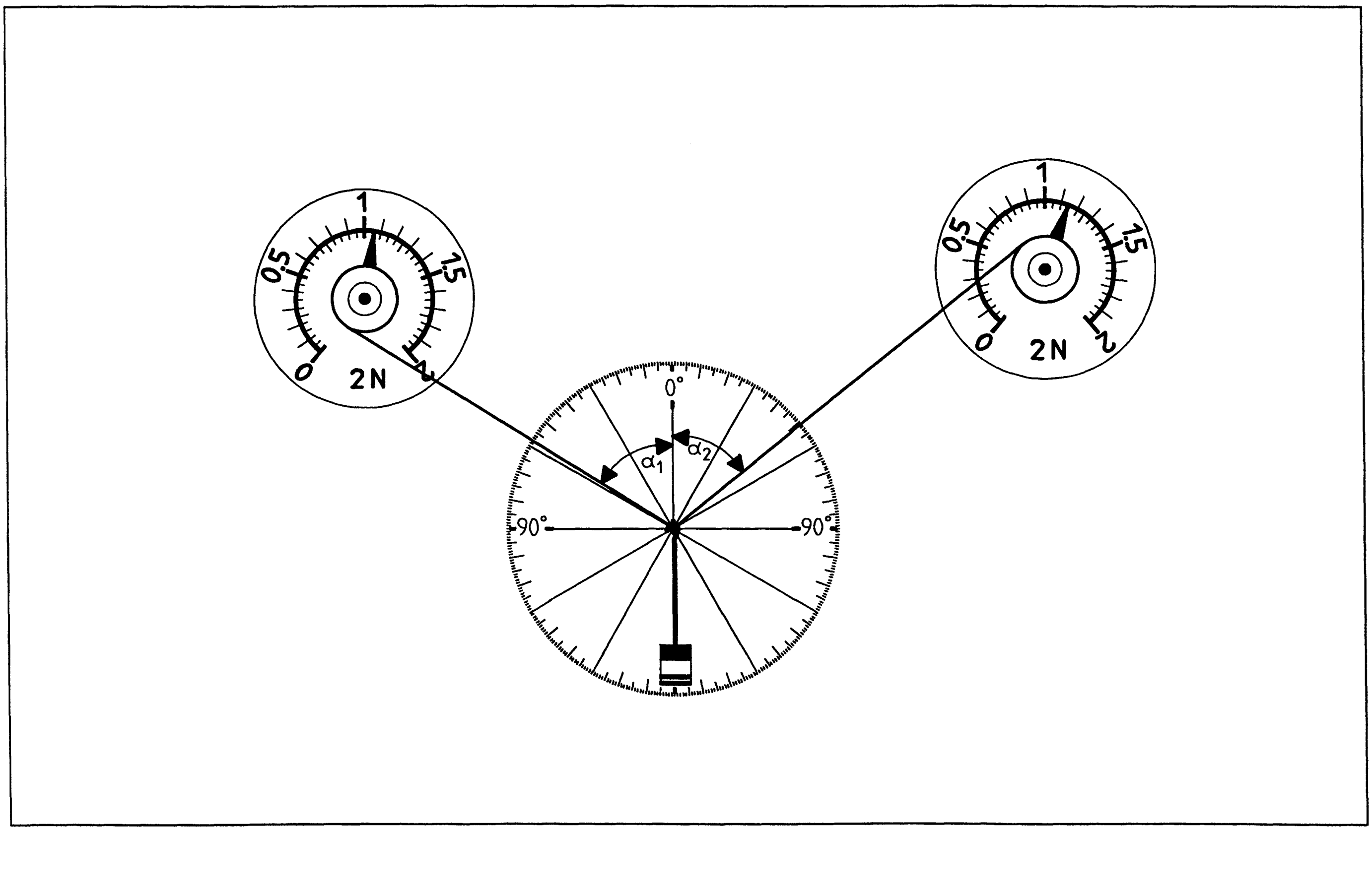
Equipment

Demonstration board for physics	02150.00	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	1
Slotted weight, 10 g, silver	02205.02	1
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	1
Protractor disk, magnet held	08270.09	1
Fish line, 0.5 m of	02090.00	1
White board pen, water soluble		

Set-up and procedure

— Place one dynamometer onto the demonstration board and adjust it.
— Attach a small loop made of fish line to the hook of the weight holder.
— Hang the weight holder with slotted weights (2 x 10 g, 1 x 50 g) on the dynamometer; read and record the force F indicated on the dynamometer.
— Place the second dynamometer onto the demonstration board, adjust it, and hook its traction cord to the point of application of force $\vec{F}$.
— Shift the two dynamometers in such a manner that their traction cords enclose an arbitrary angle between them.
— Place the protractor disk such that its centre is behind the point of application of the forces.
— Read the values indicated by the dynamometer for F_1 and F_2 and determine the angles α_1 and α_2 which $\vec{F_1}$ and $\vec{F_2}$ enclose with the perpendicular on the protractor disk.
— Record the results in Table 1.
— Change the position of the dynamometers several times, and determine the respective quantities F_1 and F_2 as well as the corresponding angles α_1 and α_2 (include the case where $\alpha_1 + \alpha_2 = 90°$). While doing so, ensure before each measurement that the point of action of the forces coincides with the centre of the protractor disk. Record the measured values in Table 1.
— Remove the two dynamometers. With the aid of the protractor disk and the scale construct the force parallelogram with the white board pen for one of the investigated cases on the demonstration board (Fig. 2).

Fig. 1



Results

$F = 1.3$ N

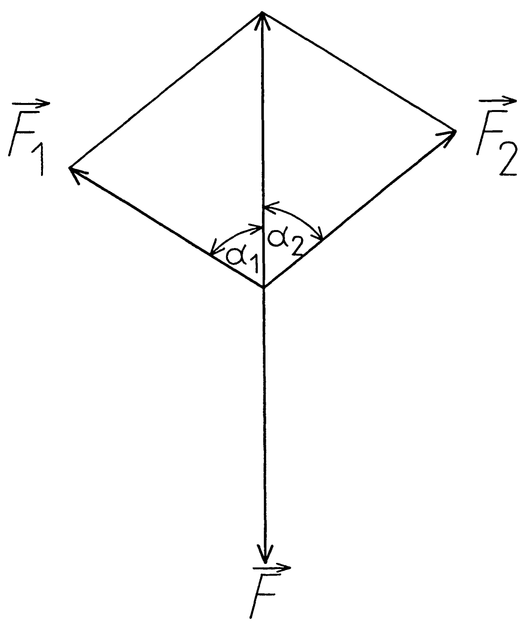
Table 1 (Sample measurements)

F_1/N	F_2/N	$\alpha_1/1°$	$\alpha_2/1°$	(F_1+F_2)/N	$(\alpha_1+\alpha_2)/1°$
1.06	1.14	58	51	2.20	109
0.66	1.11	60	30	1.77	90
1.53	1.25	52	75	2.78	127
0.78	0.79	36	33	1.57	69

Evaluation

The sum of the magnitudes of $\vec{F}_1$ and $\vec{F}_2$ is always larger than the magnitude of the force $\vec{F}$ which is to be resolved. The larger the angle enclosed by the forces ($\alpha_1 + \alpha_2$), the larger their sum is.

Fig. 2



In any case, $\vec{F}_1$ and $\vec{F}_2$ result in the same action as the force $\vec{F}$. They are termed the components of $\vec{F}$. $\vec{F}_1$ and $\vec{F}_2$ can be determined by drawing their lines of action and the force $\vec{F}$; thus constructing a force parallelogram, whose diagonal is formed by $\vec{F}$. The components $\vec{F}_1$ and $\vec{F}_2$ form the sides of the parallelogram.

A force can be resolved into components whose lines of action intersect in the point of application of the force. The components can be determined by construction or calculation.

Remarks

In this experiment the weight holder with slotted weights has been selected to preset a force which is then resolved into its components. A helical spring which has been displaced by a certain distance is also appropriate for pre-setting the force. In this case the position of the protractor disk whose centre marks the end of the extended spring may not be changed.

It is advisable to have the students simultaneously construct the same force parallelogram in their notebooks while the teacher is drawing it on the demonstration board.

The special case in which $\alpha_1 + \alpha_2 = 90°$ was selected so that the students could check their results with a sample calculation even without knowledge of trigonometry.

An additional task could be a graphical check of the remaining measurements.

Recording an exact series of measurements is not absolutely necessary. One can also restrict the experiment to a single measurement of F_1, F_2, α_1 and α_2 and determine the force parallelogram by quadrupling the values. In this case, one should however demonstrate qualitatively that the components enclose arbitrary angles and as a result can have differing magnitudes.

Resolution of forces on an inclined plane

Demonstrate that the weight of a body on an inclined plane can be resolved into two components which are perpendicular to each other and one of which acts in the direction of the slope.
Additionally, investigate how the force acting down the plane can be calculated.

Equipment

Demonstration board for physics	02150.00	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Inclined plane	02204.00	1
Scale for demonstration board	02153.00	1
Protractor disk, magnet held	08270.09	1
Roller	11301.01	1
White board pen, water soluble		

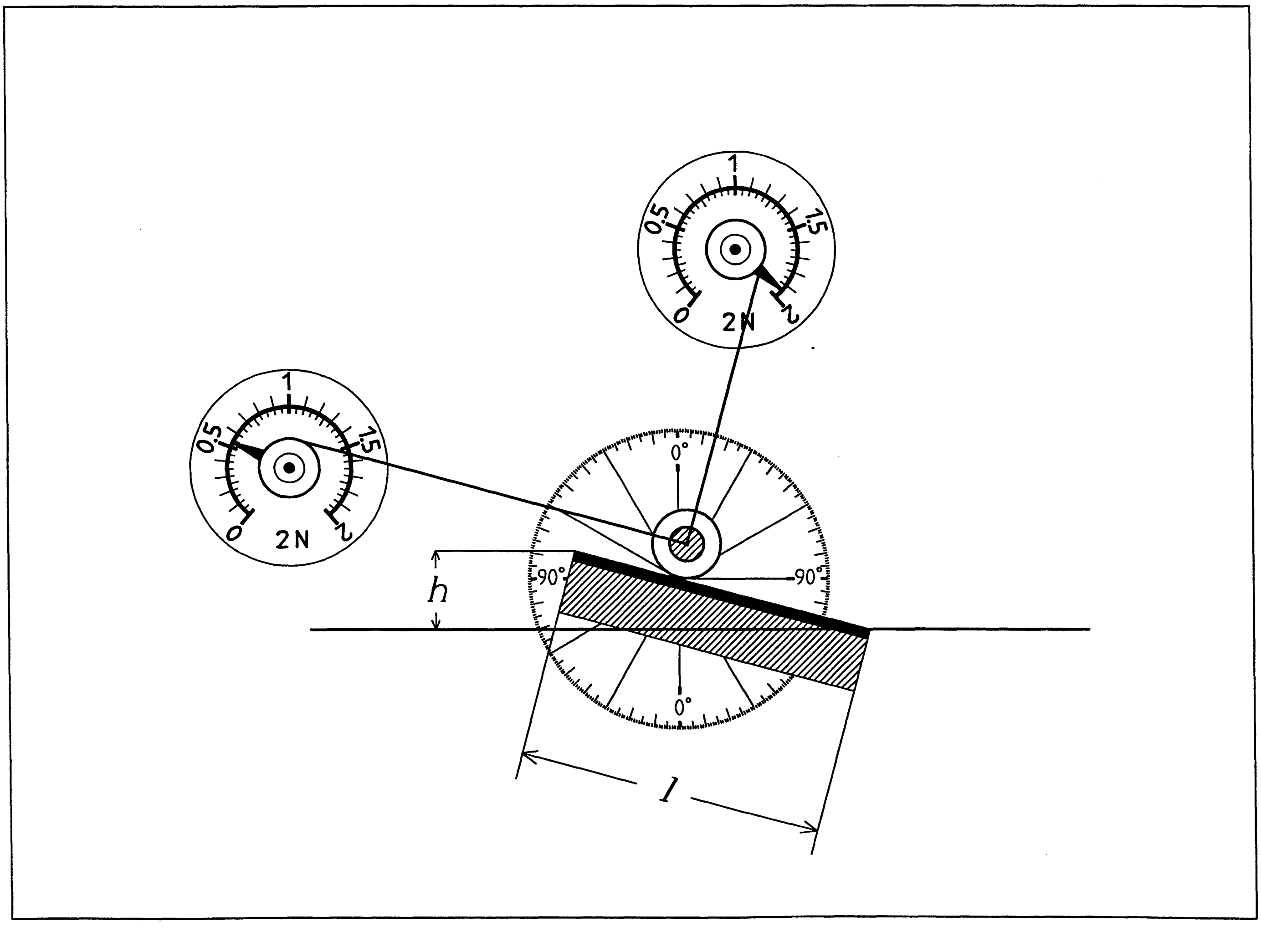
Set-up

— Place the protractor disk onto the demonstration board in such a manner that the intended vertical line (the zero degree line) is vertical.
— Draw a horizontal line on the demonstration board with the white board pen, e.g. as shown in Fig. 1.
— Place the inclined plane onto the protractor disk such that it forms an angle $\alpha = 15°$ with the horizontal line and touches the previously drawn horizontal line with its lower end.
— Locate the two dynamometers at positions similar to those indicated in Fig. 1 and adjust them.

Procedure

— Measure and record the length l of the inclined plane.

Fig. 1



— Measure the height h of the upper end of the inclined plane from the drawn horizontal line (Fig. 1) and record its value in Table 1.
— Hang the roller on the traction cord of the left dynamometer. Measure the weight F_G of the roller and record it.
— Place the roller onto the inclined plane, and move the dynamometer until its traction cord is parallel to the inclined plane.
— Also connect the right dynamometer with the roller. Shift it until the traction cord is perpendicular to the inclined plan and the roller is lifted slightly from the plane.
Note: The angle between $\vec{F}_H$ and $\vec{F}_N$ is exactly a right angle when F_N is minimal. This can be achieved by making small changes in the position of the dynamometer for F_N until a right angle has been formed.
— Record the values for F_H and F_N in Table 1.
— Increase angle α in 15° steps. Record the respective values for F_H, F_N and h in Table 1.
— For a constant angle, e.g. $\alpha = 60^\circ$, determine the values for F_H and F_N with reduced weight $\vec{F}_G$ of the roller. To achieve this, first unscrew and remove one of its supplementary weights (each 50 g); subsequently, the other one. In each case measure F_G as described at the beginning of this experiment and then determine F_H and F_N. Record the measured values.

Results

Table 1 $l = 31$ cm

F_G/N	$\alpha/1^\circ$	F_H/N	F_N/N	h/cm	F_H/F_G	h/l
2.0	15	0.53	1.94	8.1	0.26	0.26
2.0	30	1.00	1.74	15.5	0.50	0.50
2.0	45	1.40	1.40	22.0	0.70	0.71
2.0	60	1.75	1.00	26.0	0.87	0.86
1.51	60	1.30	0.75	26.8	0.86	0.86
0.99	60	0.86	0,50	26.8	0.86	0.86

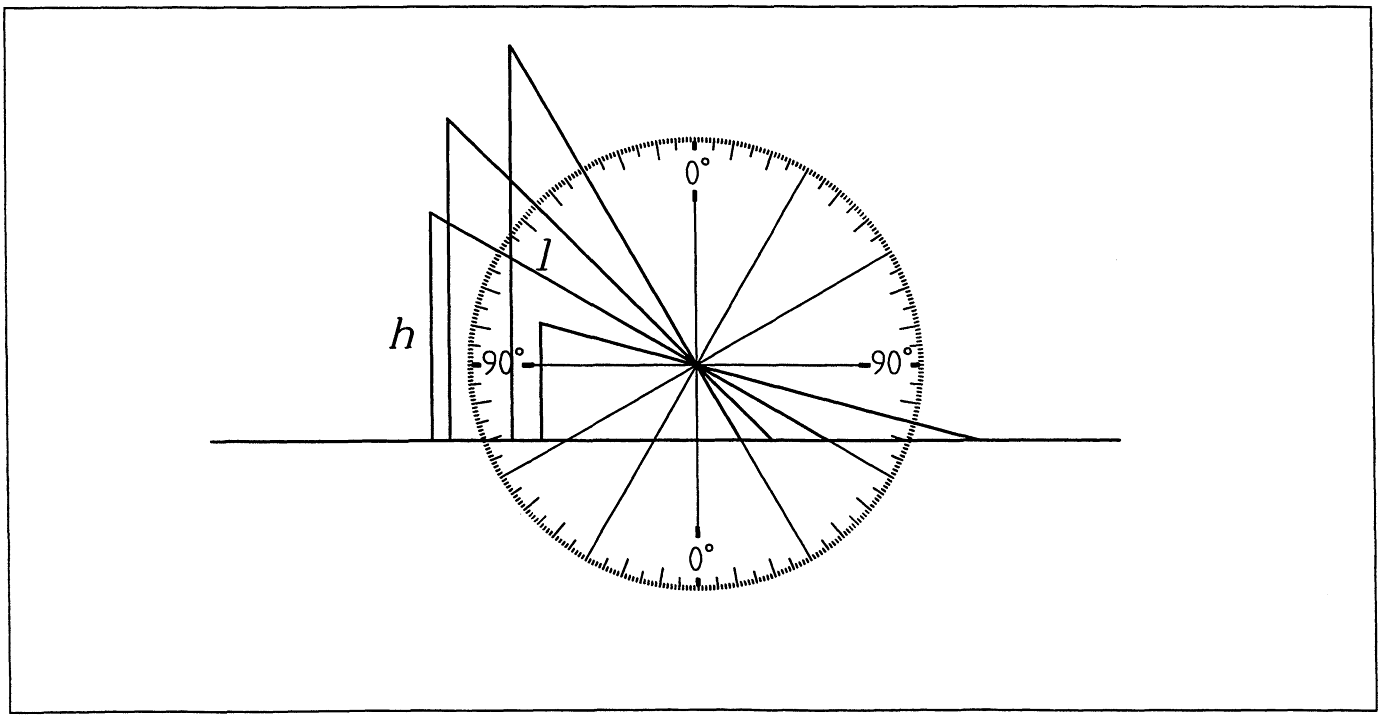
Evaluation

For a constant weight $\vec{F}_G$, the larger the angle of slope – and thus the height of the inclined plane – the larger the downslope force $\vec{F}_H$ and the smaller the perpendicular (normal) force $\vec{F}_N$.
To be able to make quantitative statements, calculate the quotient F_H/F_G, and the following is obtained (within the given limits of the measuring accuracy):
F_H/F_G = constant.
As can be seen subsequent to quotient formation (cf. Table 1), this constant has a value of h / l, therefore:

$$\frac{F_H}{F_G} = \frac{h}{l} \text{ or } F_H = F_G \cdot \frac{h}{l}.$$

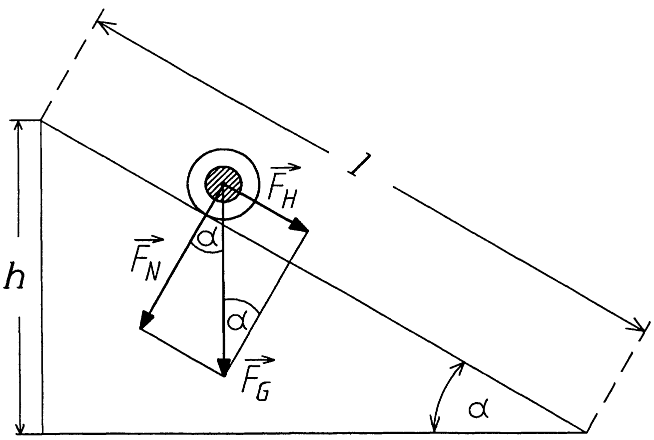
Using this equation, F_H can be calculated.

Fig. 2



The drawing on the demonstration board (cf. Fig. 2) corroborates the corresponding facts.
F_H/F_G = constant also means that $F_H \sim F_G$. This can be confirmed with the results of the last two measuring steps (cf. the last two lines in Table 1).

Fig. 3



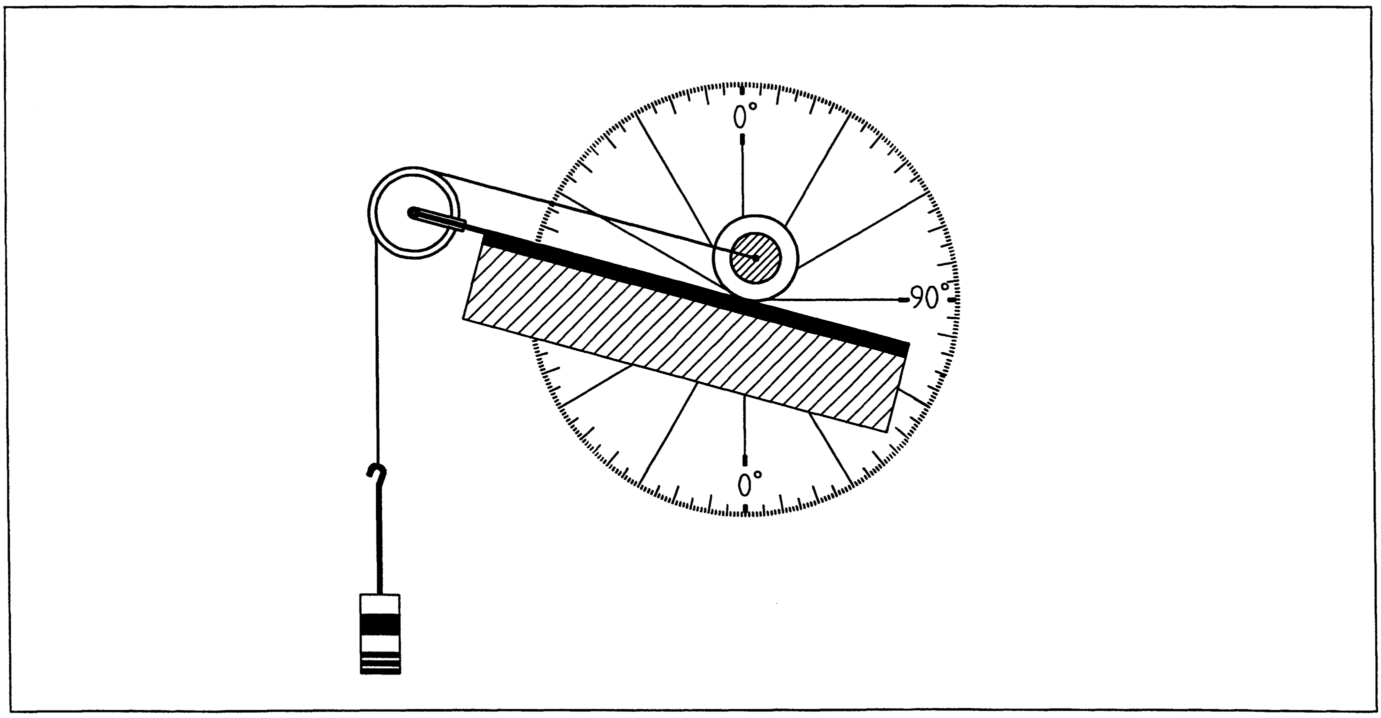
Remarks

If the students have appropriate previous knowledge of trigonometry, the result can also be written in the following form:

$$F_H/F_G = \sin\alpha \quad \text{or} \quad F_H = F_G \cdot \sin\alpha.$$

It is advisable to use the student's existing knowledge of force resolution and the similarity of triangles to allow the students to predict the result of the measurements with the aid of a corresponding sketch (Fig. 3). In this case, the experiment has now become a confirmation experiment with the advantage that the individual steps leading to the results can be taken with more clarity over the objective.
A planning drawing for the measurements, similar to that shown in Fig. 2, can be drawn on the demonstration board even before the experiment.
The correlation between the downslope force and the angle of slope can also be elaborated by using weights to preset specific values for F_H and the corresponding angle, determined. To achieve this, a roller (02262.00) is attached to the inclined plane with the help of the thumb screw.

Fig. 4

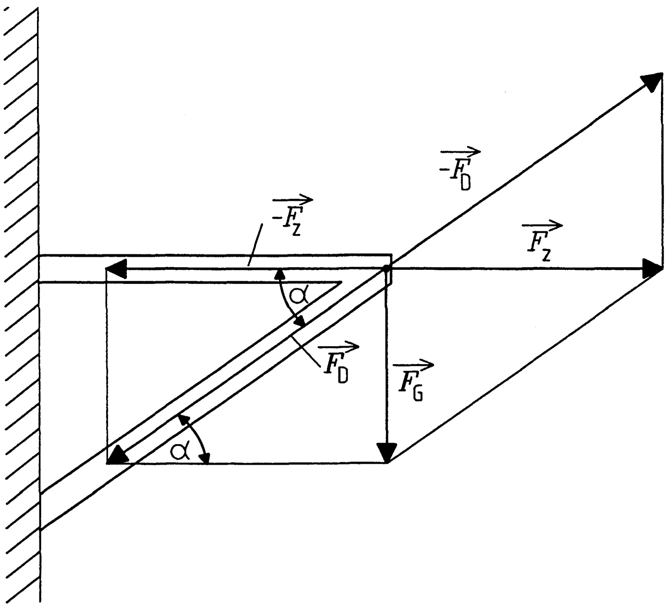


Resolution of forces on an inclined plane

Space for notes

Demonstrate which forces act on a simple crane on which a load is acting and how these forces can be determined.

Fig. 1



Equipment

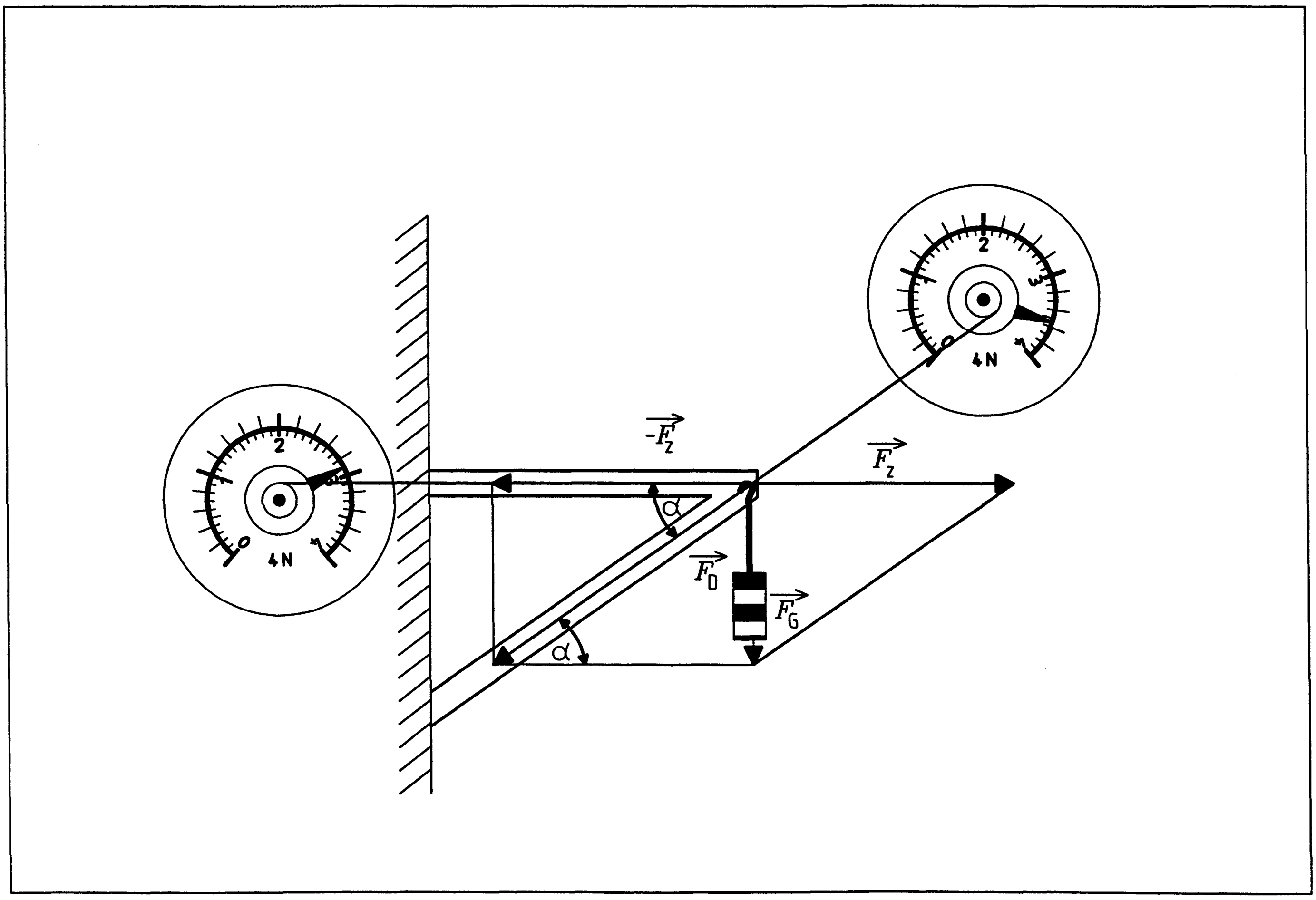
Demonstration board for physics	02150.00	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Protractor disk, magnet held	08270.09	1
Fish line, 0.5 m from	02090.00	1
White board pen, water soluble		

Set-up and procedure

For the formulation of the problem as well as in preparation for the evaluation of the experiments, make a drawing of a simple crane on the demonstration board; also discuss and draw in the forces occurring under load (cf. Fig. 1).

— Load the weight holder with 4 slotted weights.
— Attach a small loop made of fish line to the weight holder's hook.
— Use both dynamometers in their 4 N measuring ranges.

Fig. 2



- Place one dynamometer onto the demonstration board. Measure the weight F_G of the loaded weight holder, and record F_G.
- Place the other dynamometer to the right above the crane.
- Hook the traction cords of the two dynamometers together. Hang the weight holder's loop onto the joined hooks.
- Shift the dynamometers until their traction cords lie along the lines of action of $\vec{F}_Z$ and $\vec{F}_D$ and the common point of application of the forces $\vec{F}_G$, $\vec{F}_D$ and $\vec{F}_Z$ coincide with the point of application marked on the demonstration board (cf. Fig. 2).
- Measure and record F_Z and F_D.
- Change angle α; whereby the point of application remains unchanged. In each case observe the dynamometer. Record your observations.
- Remove the dynamometers and the weight holder.
- Place the protractor disk onto the demonstration board in such a manner that its centre coincides exactly with the forces' point of application.
  Note: To do this, extend the lines of action of the forces $\vec{F}_G$, and $\vec{F}_Z$ sufficiently far that they are not completely covered by the protractor disk.
- Measure the angle α (cf. Fig. 1), which is enclosed by the lines of action of $\vec{F}_Z$ and $\vec{F}_D$.

Results (sample measurement)

F_G = 2.04 N
F_Z = 2.90 N
F_G = 3.55 N
α = 35°

Observation: The smaller the angle α, the larger the corresponding tractive and compressive forces.

On a loaded crane, the boom is subjected to compression and (for a boom with three degrees of freedom) the load cable to traction. The sum of the magnitudes of the occurring forces is always greater than the magnitude of the force exerting the load:

$$F_Z + F_D > F_G.$$

How large F_Z and F_D have to be in order for an equilibrium to exist depends on the angle α which the forces $\vec{F}_Z$ and $\vec{F}_D$ enclose. The students can predict he observed correlation between α, F_Z, and F_D (for constant F_G).
To check the measured values the corresponding force parallelogram is drawn to scale on the demonstration board.

Remarks

If the students have appropriate trigonometric knowledge, the experimental results can not only be graphically but also mathematically evaluated:

$$\sin\alpha = F_G/F_D \rightarrow F_D = F_G/\sin\alpha;$$
$$\tan\alpha = F_G/F_Z \rightarrow F_Z = F_G/\tan\alpha \text{ (cf. Fig.1)}$$

With the concrete measured values one obtains the following:

F_D = 2.04 N/sin 35° = 3.56 N;
F_Z = 2.04 N/tan 35° = 2.91 N.

If the students have the above-mentioned knowledge, the values for F_z and F_D could also be calculated in advance. Then the experiment would have confirming character.

Restoring force on a displaced pendulum

Using a thread pendulum, investigate how large the force with which the displaced pendulum approaches its resting position is.
In addition, investigate how this force, which is termed the restoring force, depends on the angle of displacement of the pendulum and how one can calculate it.

Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver	02205.02	2
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Protractor disk, magnet held	08270.09	1
Fish line, 1 m from	02090.00	1
White board pen, water soluble		

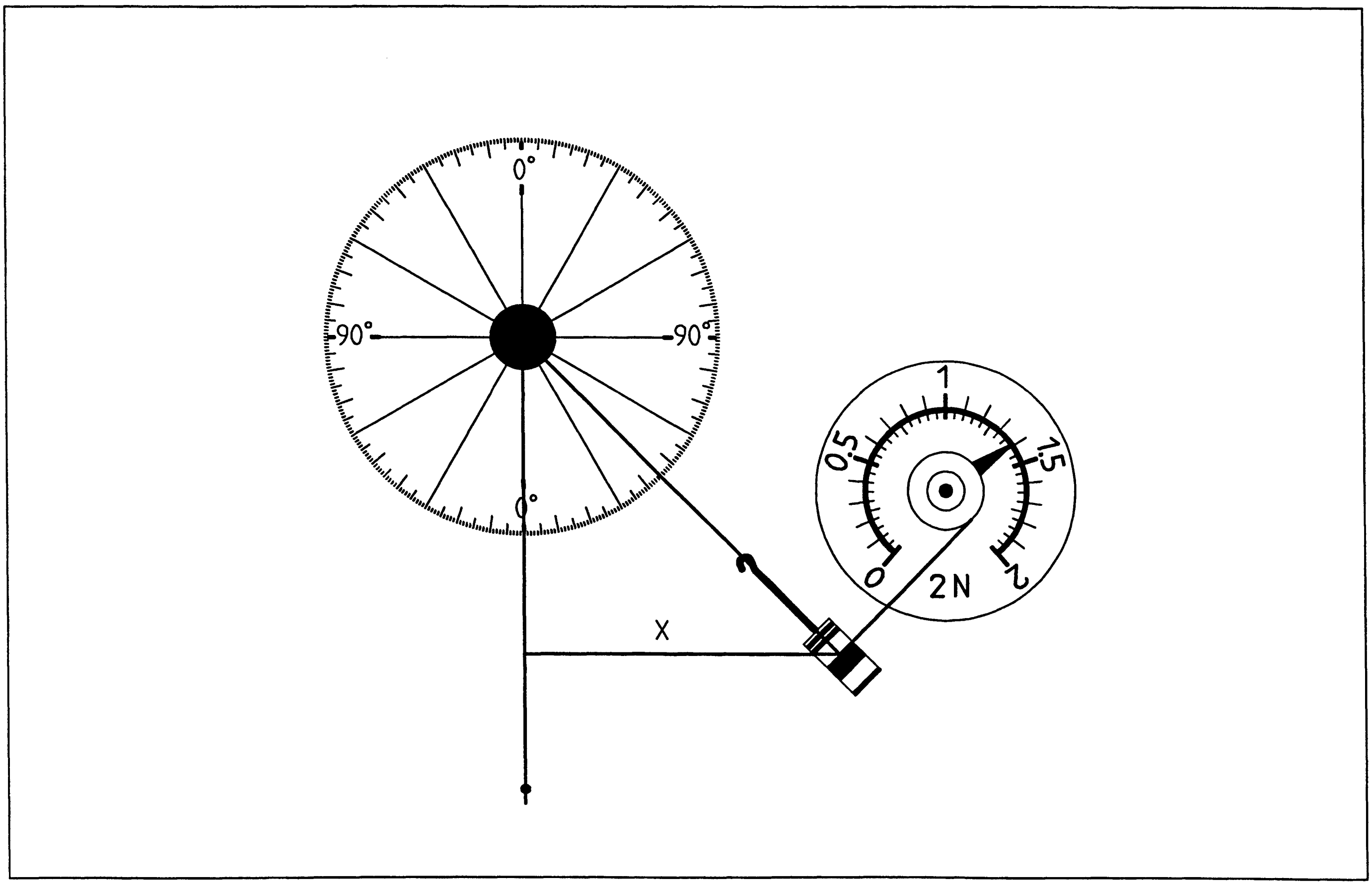
Set-up

— Prepare a piece of fish line which is approximately 40 cm long and has two terminal loops.
— Arrange the slotted weights on the weight holder as shown in Fig. 1. Determine the centre of gravity of this body which is intended to be the pendulum bob (it lies approximately between the second and the third 50-g slotted weight, starting from the weight holder).
— Place the protractor disk onto the demonstration board in such a manner that the line $\overline{00}$ is vertical and the disk extends beyond the upper edge of the board.
— With the white board pen extend the line $\overline{00}$ almost to the lower edge of the board.
— Place the axle on fixing magnet in the centre of the protractor disk.

Procedure

— Measure and note the weight F_G of the pendulum bob.
— Hang the pendulum bob on the axle with the prepared cord.
— Mark the position of the pendulum bob's centre of gravity on the vertical line on the demonstration board (Fig. 1).
— Measure and record the length of the pendulum l.

Fig. 1



— Displace the pendulum and allow it to oscillate; note your observation.
— Stop the pendulum.
— Place the dynamometer onto the demonstration board and hook the traction cord at the centre of gravity of the pendulum bob.
— Set the dynamometer to 0 and shift it until the pendulum has been displaced by 15° and the traction cord of the dynamometer forms a right angle with the pendulum's thread.
— Mark the position of the centre of gravity of the displaced pendulum bob on the board.
— Read the indicated force F_r; then measure the distance x between the pendulum bob's centre of gravity and the drawn perpendicular line. Record F_R and x in Table 1.
— Repeat the measurements in steps of 15° each and record the measured values.

Results

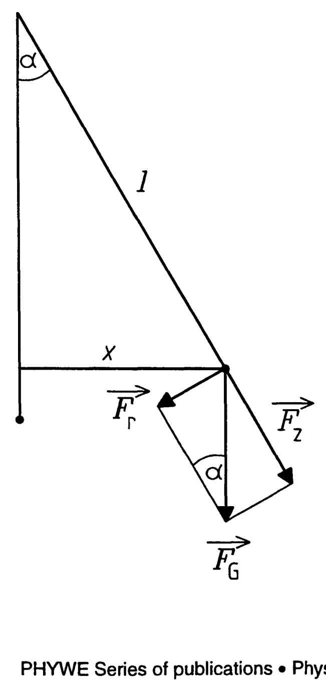
F_G = 1.94 N
l = 48.5 cm

Observation: After the displaced pendulum is released, it oscillates around its resting position.

Table 1 (sample measurement)

α / 1°	F_r / N	x / cm	F_r / F_G	x / l
15	0.51	12.6	0.26	0.26
30	0.97	24.0	0.50	0.49
45	1.39	34.2	0.72	0.71
60	1.65	42.0	0.85	0.87

Fig. 2



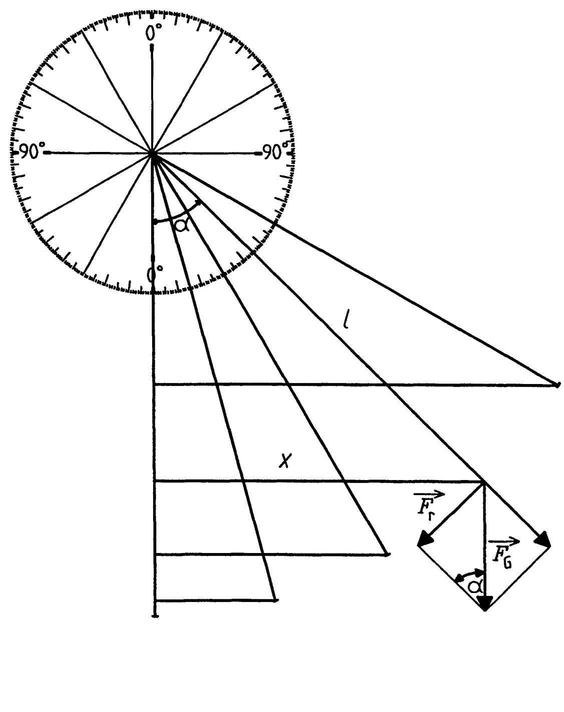
Evaluation

The pendulum which has been displaced to the right is accelerated to the left by the restoration force $\vec{F_r}$ back to its resting position; due to its inertia it swings beyond this position. The restoration force is equal to zero in the resting position, then however it again becomes larger and swings the pendulum to the right in the direction of the resting position.
As the students can predict, the larger the distance x or the angle α is, the larger F_r is. This is confirmed by the measurements.
The calculation of the quotients $F_r/F_G = x/l$ (cf. Table 1) demonstrates the following: $F_r/F_G = x/l$ or $F_r = F_G \cdot x/l$.
This result is corroborated by the completion of the diagram on the demonstration board (cf. Fig. 2 and Fig. 3). In the process, the student's knowledge of the similarity of triangles is used.

Remarks

If the students have knowledge of trigonometry, then the result can also be written in the form $F_r/F_G = \sin\alpha$ or $F_r = F_G \cdot \sin\alpha$. And in this case, the measurement of x can be omitted, if desired.
In connection with the discussion of the relationship between F_r and F_G, the second component of $\vec{F_G}$ should also be interpreted: it is a tractive force (in Fig. 2 designated $\vec{F_Z}$), which spans the pendulum.

Fig. 3



Determination of the centre of gravity of an irregular plate

The centre of gravity of an irregularly delimited plate with the aid of several gravity lines (plumb lines).

Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	1
Centre-of-gravity plate	02300.01	1
Fish line, 1 m of	02090.00	1
White board pen, water soluble		

Set-up and procedure

— Place the axle on fixing magnet onto the demonstration board.

— Hang the centre-of-gravity plate on the demonstration board from any arbitrary point. Avoid using the point marked with Δ (cf. Fig. 1) to begin with.

— Tie loops in both ends of a piece of fish line which is approximately 20 cm in length, and hang the weight holder with slotted weight on the axle as plumb bob (Fig. 1).

— Mark the course of the line on the centre-of-gravity plate with the white board pen.

— Allow the centre-of-gravity plate to oscillate and observe it.

— Hang the centre-of-gravity plate on at least one additional point and proceed as above.

— Record your observations (1).

— Remove the centre-of-gravity plate from the axle and extend the gravity lines.

— Formulate your results (2).

— Hang the centre-of-gravity plate from the intersection point of the lines (point Δ), and move it into different positions by rotating it; formulate your observation (3).

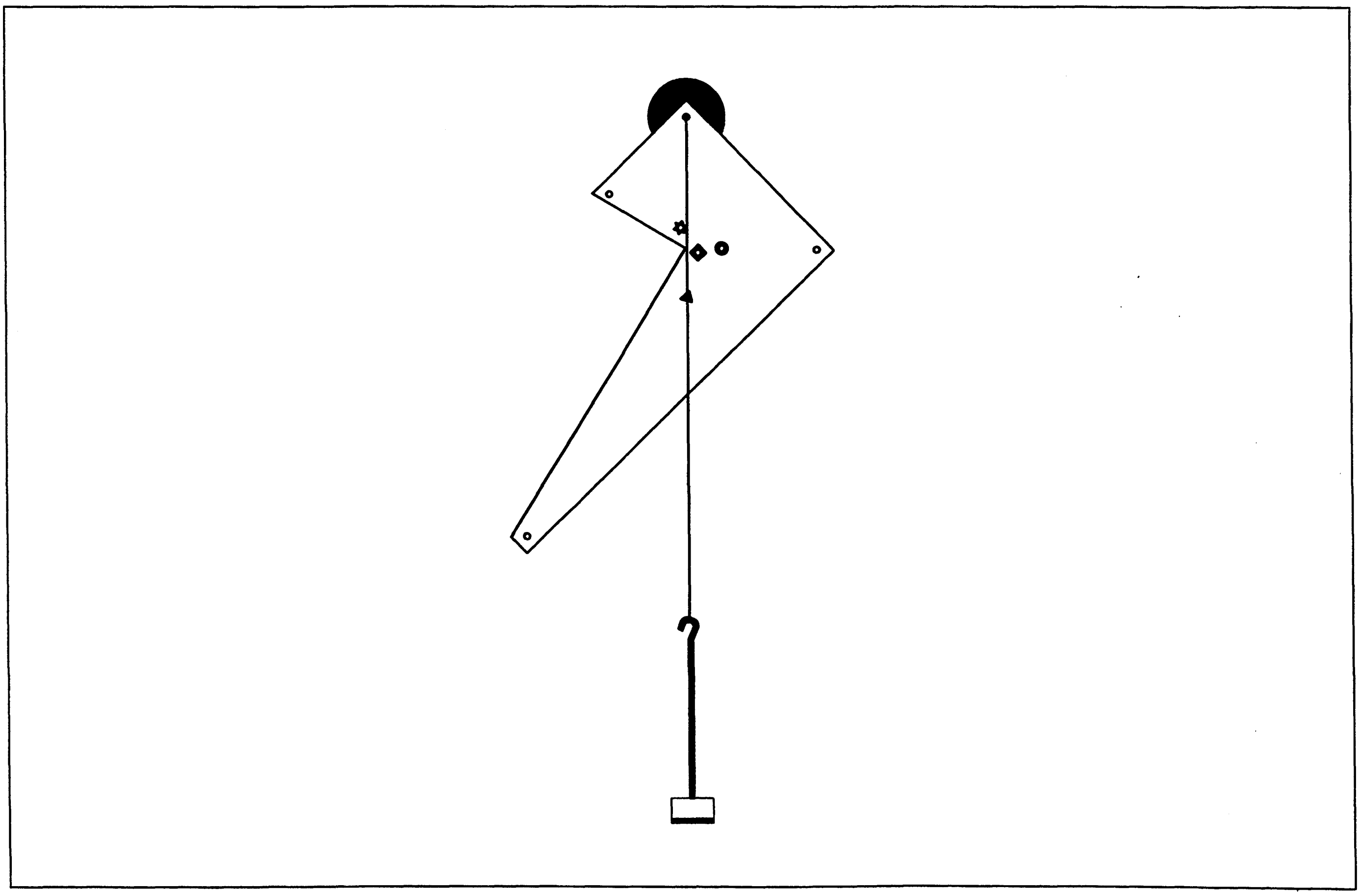
— Hang the centre-of-gravity plate from additional points and while doing so observe the position of the centre of gravity; formulate your observation (4).

Results

(1) After being displaced the plate always oscillates back to its respective resting position.

(2) The plumb lines all intersect in a single point.

Fig. 1



(3) When the plate is hung on this point, it remains at rest in any arbitrary position after being displaced.
(4) If the plate is not hung from the intersection point of the plumb lines, their intersection point is always vertically below the suspension point.

Evaluation

The intersection of the gravity lines (plumb lines) of a body is the centre of gravity of this body.
The centre of gravity can be determined by freely suspending the body from (at least) two arbitrary points, by drawing the plumb lines and determining their intersection point.

Remarks

One can also use the results of this experiment to make the students aware of the different types of equilibria: stable, labile and indifferent equilibrium are then present when the centre-of-gravity plate is suspended above, below or on its centre of gravity.
The comprehension of the concept of the centre of gravity can be further increased if one performs balancing experiments with the plate or allows them to be conducted by the students.

Frictional force

Demonstrate that sliding friction is:
— Smaller than static friction
— Proportional to the normal force,
— A function of the nature of the contact surfaces, and
— Independent of the size of the contact surface.

Equipment

Demonstration board for physics	02150.00	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Plane	02152.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Friction block	02240.01	1
Holding pin	03949.00	1
Spirit level or box spirit level		

Set-up

— Place the adjustable surface (plane) onto the demonstration board and ensure that it is horizontal.
— Position the dynamometer at approximately the same height and set it to zero.
— Measure the weight F_G of the friction block and record it
— Place the friction block with its rubberised surface downwards onto the plane.
— Hook the traction cord of the dynamometer onto the friction block's hook and shift the position of the dynamometer until the traction cord is parallel to the plane (Fig. 1).

Procedure

— Move the dynamometer slowly, but as uniformly as possible to the right; observe the deflection of the needle and the movement of the block.
— If necessary, repeat the procedure several times; record your observation.
— Using the holding pin, load the friction block successively with 1, 2 and 3 50-g slotted weights thus increasing F_G. Measure the sliding friction F_r for the respective weights (if necessary, use the 4-N measuring range) and record the values in Table 1.
— Remove the slotted weights and the holding pin.
— First, lay the friction block with its largest non-rubberised surface on the plane, then with the intermediate non-rubberised one downwards. Measure F_r in both cases as above. Record the values.

Results

$F_G = 0.86$ N

Observation: At the beginning of the movement of the friction block the frictional force is larger than during its uniform movement. Sample measurement: The dynamometer needle is initially deflected by approximately 0.7 N and subsequently remains steady at approximately on 0.6 N

Fig. 1

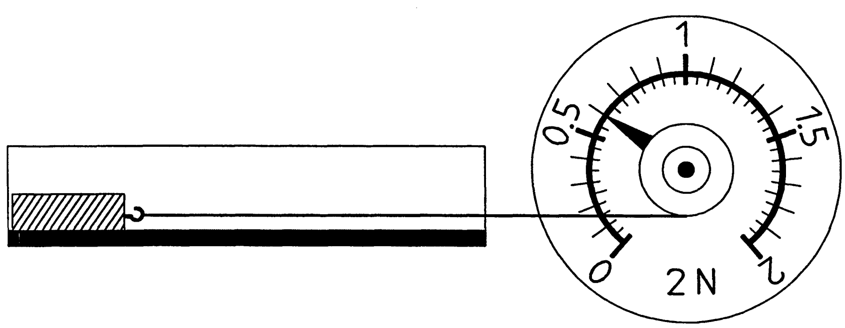


Table 1 (Sample measurement)

Contact surface	F_G / N	F_r / N	F_r / F_G
Large, rubber	0.86	0.60	0.70
	1.42	0.91	0.64
	1.92	1.36	0.71
	2.42	1.75	0.72
	2.92	2.15	0.74
Large, wooden	0.86	0.12	0.14
Intermediate, wooden	0.86	0.12	0.14

Evaluation

In order to move bodies, one of which lies upon the other, uniformly a force is required which overcomes the frictional force F_r. The force which opposes the movement results from the fact that the surfaces of the bodies which are in contact with each other "interlock". Interlocking can not take place to the same extent during movement as at rest. Therefore, the frictional force is larger at the instant of the onset of movement as during movement.

This corresponds to the observation, and formulated as a concept, it is stated as follows: The sliding frictional force $\vec{F}_r$; is smaller than the static frictional force $\vec{F}_h$;

$$F_r < F_h.$$

As Table 1 (upper part) shows, the frictional force increases with the weight F_G. If one forms the quotient F_r/F_G, one obtains, within the limits of the measurement accuracy, a constant, which is termed the coefficient of friction μ and which has a mean value of 0.7 under the given experimental conditions.

Therefore:

$$F_r = \mu \cdot F_G.$$

In this experiment $\vec{F}_G$ acts perpendicularly to the sliding surface. If the sliding surface is not horizontal, then only a component of $\vec{F}_G$, the normal force $\vec{F}_N$, is decisive for the sliding friction. Consequently, the following is valid in the general case:

$$F_r = \mu \cdot F_N.$$

A comparison of the measured values in Table 1, first and last lines, shows that the frictional force F_r is dependent on the nature of the contact surfaces.

A comparison of the measured values in the last two lines of Table 1 shows that the frictional force is independent of the magnitude of the contact surfaces.

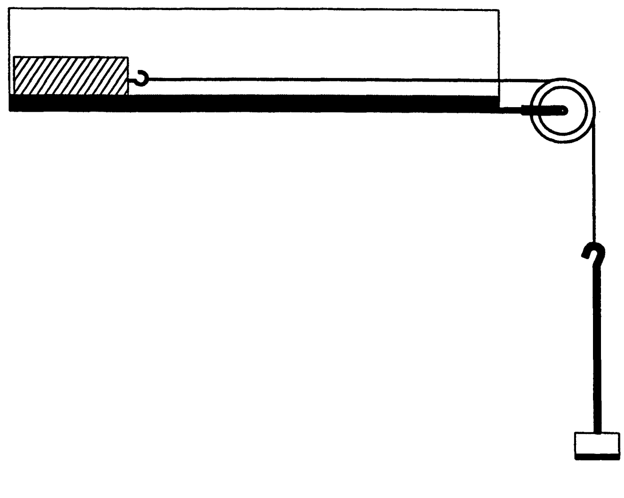
Remarks

This experiment can be expanded by scattering sand onto the surface of the inclined plane. Or one can cut a strip of sand paper or other material, place it onto the plane and secure it against slipping with the thumbscrew. In this manner, additional coefficients of friction can be determined.

These experimental results are only a good approximation. In reality there are many factors which require special consideration. Thus, on auto tyres having a sophisticated tread, the width of the contact surfaces may have a great influence on the sliding friction and thus on the roadability (e.g. the tyres of race cars).

The frictional force F_r can also be determined by progressively increasing the force acting horizontally on the block by adding weights and by pushing the block until a movement of the block with constant velocity results. To achieve this, a roller (03970.00) is attached to the inclined plane using the thumb screw (Fig. 2); in addition, 1-g weights (03916.00) will generally be required.

Fig. 2



Determination of the coefficient of friction on an inclined plane

Demonstrate that one can determine the coefficient of friction μ on an inclined plane without measuring any forces.

Equipment

Demonstration board for physics	02150.00	1
Plane	02152.00	1
Scale for demonstration board	02153.00	1
Slotted weight, 50 g, black	02206.01	2
Friction block	02240.01	1
Holding pin	03949.00	1
Protractor disk, magnet held	08270.09	1
White board pen, water soluble		

Set-up and procedure

— Place the protractor disk onto the demonstration board.

— Attach the plane on the protractor disk in such a manner that its lower edge passes through the centre of the protractor disk.

— Beginning with a slight angle of inclination α between the plane and the horizontal line on the protractor disk, place the friction block onto the inclined plane with its larger wooden surface downwards (Fig. 1).

— Increase the size of the angle of slope α gradually, in such a manner that the plane rotates around the centre of the protractor disk.
Read the angle α_h which is required so that the contact frictional force $\vec{F}_H$ is just overcome and the block begins to slide; read α_h.

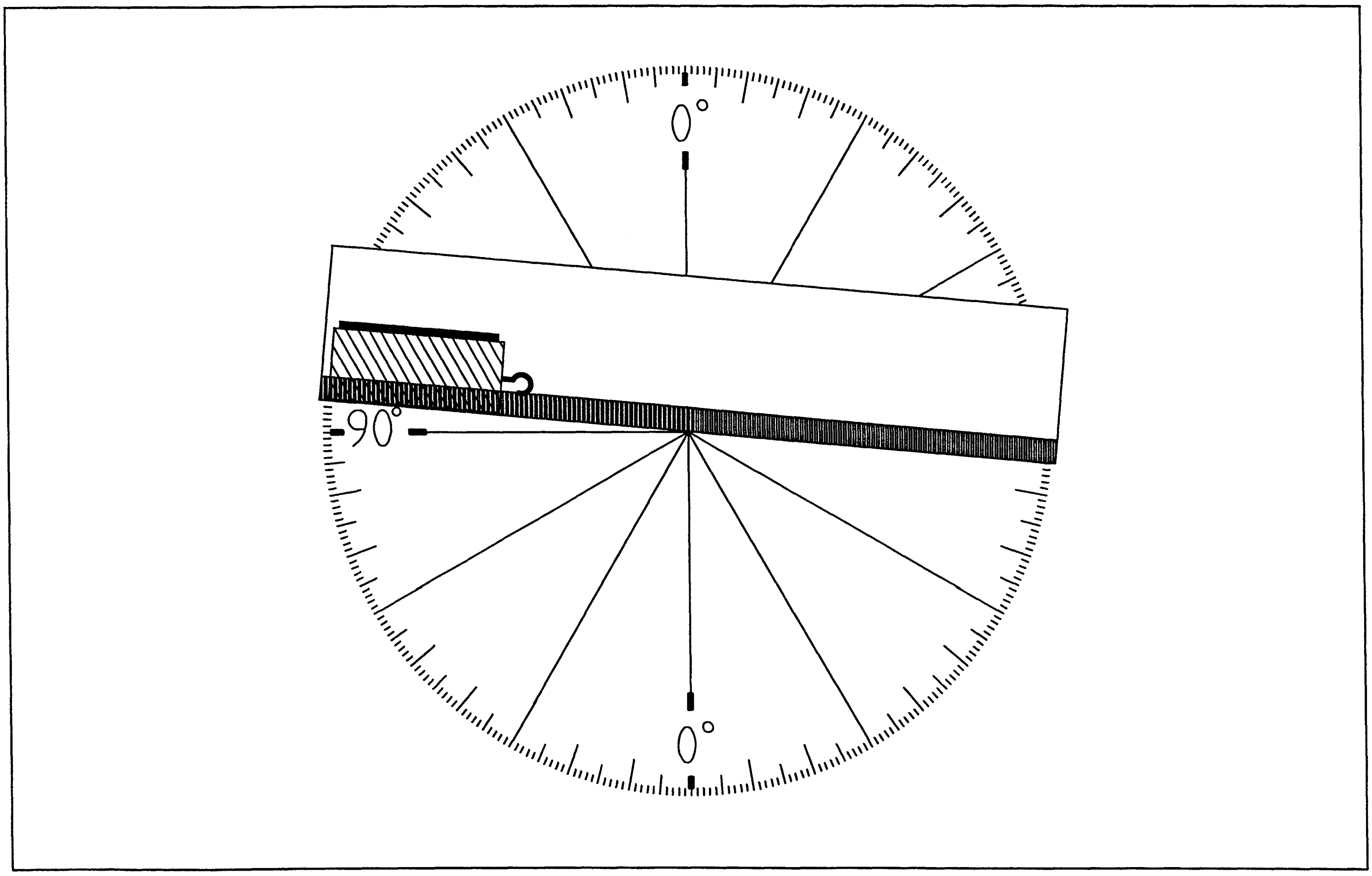
— Without changing the angle α_h place the block onto the inclined plane several times and carefully observe its movement.

— Record α_h and your observations (1).

— Return the inclined plane to its initial position, increase the angle of slope α as before, but now before the angle α_h is reached, determine the angle α at which the friction block slides down the inclined plane after being pushed lightly by trial and error; observe the block carefully; note the value of α (in Table 1) and your observation (2).

— With the aid of the holding pin, first load the friction block with one of the slotted weights, then with both of them. Determine the angle α in each case and record it.

Fig. 1



Results

(1) $\alpha_h = 11°$

Observation: After overcoming the contact frictional force, the block slides increasingly rapidly down the inclined plane.

(2) Table 1

Friction block	α /1°
Without load	10
Loaded with 50 g	10
Loaded with 100 g	10

Observation: The block slides – whether unloaded or loaded – down the slope with uniform movement after being pushed lightly

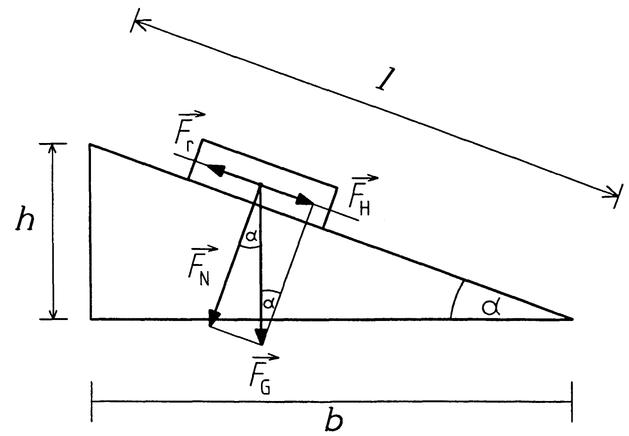
Evaluation

The following is true for the inclined plane: $F_H = F_G h/l = F_G \sin\alpha$ (cf. Fig. 2).

If a body slides down the inclined plane uniformly (with constant velocity), the movement against the opposing frictional force $\vec{F}_r$, which equal to the downslope driving force $\vec{F}_H$ in magnitude. Therefore, the following is valid:

$$\vec{F}_H = \vec{F}_r = F_G \cdot \sin\alpha\,.$$

Fig. 2



In addition, the following is true for the (sliding) frictional force:

$$\vec{F}_r = \mu \cdot \vec{F}_N\,,$$

where $\vec{F}_N$ indicates the normal force, which acts perpendicular to the inclined plane as a component of the weight $\vec{F}_G$.

Consequently, $F_G \cdot \sin\alpha = \mu \cdot F_G \cdot \cos\alpha$ (cf. Fig. 2). From this it follows that $\mu = \tan\alpha$, a result which generally surprises the student because, according to it, μ is not a function of the normal force.

With the equation $\mu = \tan\alpha$, an easily applied measuring equation for the determination of the coefficient of friction is available. The last part of the experiment, which should only serve to confirm the equation $\mu = \tan\alpha$, establishes that μ is independent of the weight $\vec{F}_G$ and thus of the normal force $\vec{F}_N$.

The results of the first part of the experiment show that the static frictional force is larger than the sliding frictional force since $F_H > F_r$. And because the downslope driving force is larger in this case than that required to compensate the sliding frictional force, the movement is accelerated.

Remarks

If the students do not have any previous trigonometric knowledge, then h and b can be measured and as a result of $F_H/F_G = h/b$ (cf. Fig. 2), the relationship $\mu = h/b$ instead of $\mu = \tan\alpha$ can be used to determine μ. Logically, this changes nothing in the recognition that μ is independent of F_N.

While turning the inclined plane, one must avoid jerky changes. Otherwise, the determination of the angle α_h and α becomes more difficult. If one changes the position of the plane progressively (each step 1°) in the vicinity of α_h and α, and in each case replaces the frictional block, the experimental procedure is usually facilitated.

If one desires to have larger angles α_h and α, the block must be placed with its rubberised side downwards.

Demonstrate that a double-sided lever is in equilibrium when the product of the applied force and the force arm is the same on both sides.

Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver	02205.02	2
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Lever	03960.00	1
Pointer for demonstration lever	03963.00	1
White board pen, water soluble		

Set-up and procedure

— Place the axle on fixing magnet onto the upper part of the demonstration board and slip the lever's middle hole onto the axle.

— Using the white board pen, draw a vertical line downwards from the axle.

— Attach the pointer for demonstration lever to the lever (in the following its tip lies directly on the vertical line when the lever is in equilibrium).

— Place the dynamometer onto the board, and measure the weight – in the following termed F_1 – for the weight holder and all of the slotted weights. Enter F_1 in the upper part of Table 1.

— Hook the traction cord of the dynamometer in the hole at the #10 index mark on the right side, and the weight holder with the slotted weights at #10 index mark on the left side.

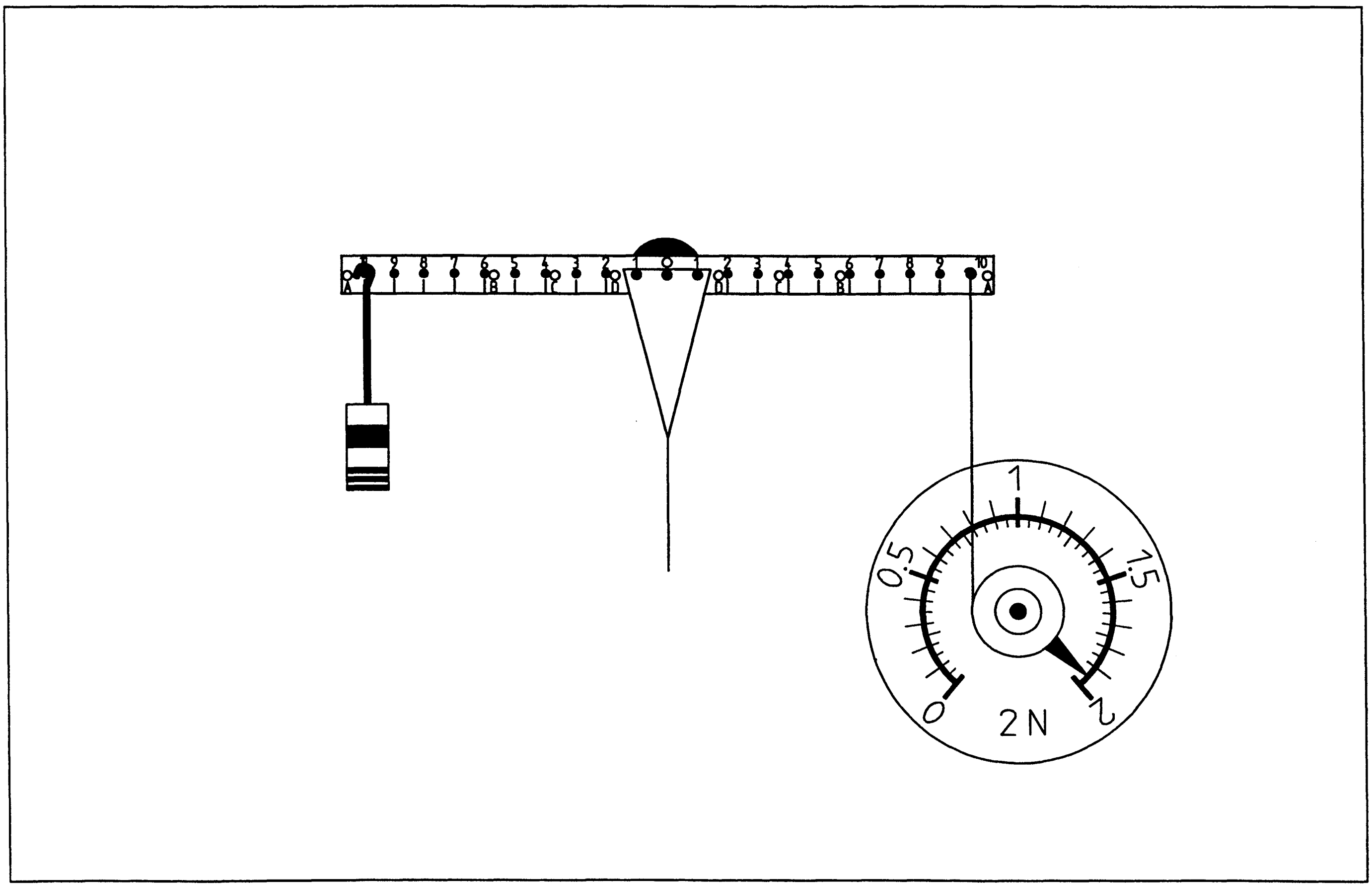
— Move the dynamometer until the lever is horizontal and the traction cord is perpendicular to it (Fig. 1).

— Read F_2 on the dynamometer and record its value in Table 1.

— Shorten the power arm l_1 progressively and in each case measure the force F_2 required for the maintenance of equilibrium and record it (cf. given values in Table 1, upper part).

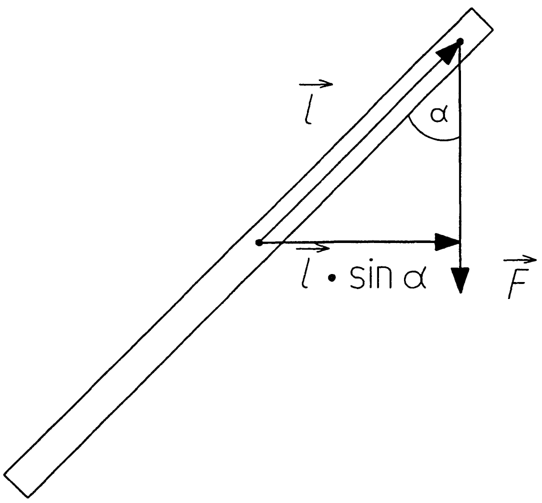
— Remove two 50-g slotted weights from the weight holder, measure the weight F_1 and record it.

Fig. 1



- Hook the weight holder into the hole at the #9 index mark, and leave it there in the following steps.
- Progressively shorten the power arm l_2. To do this, hook the traction cord of the dynamometer into the holes at the #10, #9, ..., #6 index marks (cf. Table 1, lower part), Measure F_2 in each case and record the respective values.

Fig. 2



Results

Evaluation

The distances between the index marks is 2 cm. Record the resulting l_1 and l_2 in Table 1.
After calculation of the products $F_1 \cdot l$, the following can be seen:

$$F_1 \cdot l_1 = F_2 \cdot l_2.$$

A double-sided lever is in equilibrium when the product of the forces acting to the left and to the right of the fulcrum and their respective power arms are equal. In this context, the distance between the points of application of the forces and the fulcrum of the lever is termed the power arm.

Remarks

The F_1 forces were chosen to utilise nearly the entire measuring range of the dynamometer. The pointer for the demonstration lever suggests the association with an application of the lever, the beam balance. If one does not want to bring this association to the students' minds at this time, ignore it as this point and draw a horizontal line along the lower edge of the lever.
If the term torque can be introduced at this time, generalise the determined law to the momentum theorem:

Table 1

Left side of the lever				Right side of the lever			
Index mark no.	l_1 / cm	F_1 / N	$\frac{F_1 \cdot l_1}{N \cdot cm}$	Index mark no.	l_2 / cm	F_2 / N	$\frac{F_2 \cdot l_2}{N \cdot cm}$
10	20	1.95	39.0	10	20	1.96	39.2
8	16	1.95	31.2	10	20	1.55	31.0
6	12	1.95	23.4	10	20	1.19	23.8
4	8	1.95	15.6	10	20	0.79	15.8
2	4	1.95	7.8	10	20	0.39	7.8
9	18	0.98	17.6	10	20	0.88	17.6
9	18	0.98	17.6	9	18	0.99	17.8
9	18	0.98	17.6	8	16	1.10	17.6
9	18	0.98	17.6	7	14	1.27	17.8
9	18	0.98	17.6	6	12	1.47	17.6

On a double-sided lever, the sum of all the moments of torque is equal to zero at equilibrium. For two forces, the following is true:

$$\vec{M} = \vec{F_1} \times \vec{l_1} + \vec{F_2} \times \vec{l_2} = 0,$$

where the clockwise and counterclockwise torques have different signs:

$$\vec{F_1} \times \vec{l_1} = -\vec{F_2} \times \vec{l_2}.$$

For the case in which $\vec{F}$ and $\vec{l}$ form an angle of 90°, the following is true:

$$|\vec{M}| = M = |\vec{F} \times \vec{l}| = F \cdot l,$$

However, the general case is as follows:

$$M = F \cdot l \cdot \sin \alpha.$$

$l \cdot \sin \alpha$ is also termed the effective length of the power arm or the effective lever length (cf. Fig. 2). The unit of M is the newton-meter (Nm), i.e. the same unit used to express mechanical work. This correspondence is often a source of irritation for the students.

Double-sided lever

Space for notes

One-sided lever

Prove that an equilibrium exists on a one-sided lever when the products of two conversely acting forces and their power arms are equal.

Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver	02205.02	2
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Lever	03960.00	1
Pointer for demonstration lever	03963.00	1
White board pen, water soluble		

Set-up and procedure

— Place the axle on fixing magnet onto the lower part of the demonstration board, and slip the middle hole of the lever onto the axle.

— Using the white board pen, draw a vertical line downwards from the axle.

— Attach the pointer for demonstration lever to the lever (in the following its tip lies directly on the vertical line when the lever is in equilibrium).

— Place the dynamometer onto the board, and measure the weight – in the following termed F_1 – for the weight holder with all the slotted weights. Note F_1 in the upper part of Table 1.

— Hook both the traction cord of the dynamometer and the weight holder with the slotted weights in the hole at the #10 index mark on the right side.

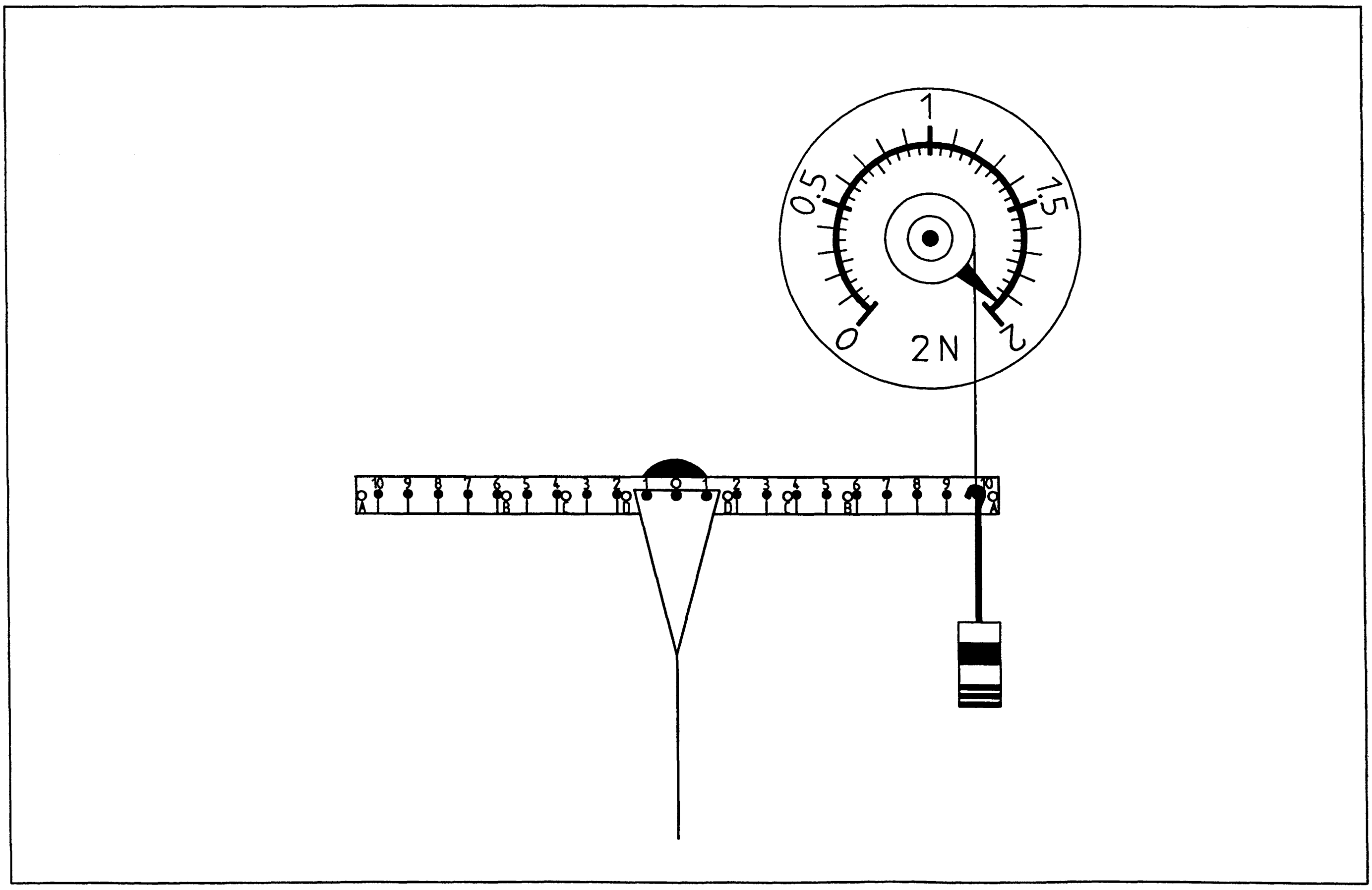
— Move the dynamometer until the lever is horizontal and the traction cord is perpendicular to it (Fig. 1).

— Read F_2 on the dynamometer, and record its value in Table 1.

— Shorten the power arm l_1 progressively. For each length measure the force F_2 required for the maintenance of equilibrium and record it (cf. given values in Table 1, upper part).

— Remove two 50-g slotted weights from the weight holder, measure the weight F_1 again and record it.

Fig. 1



— Hook the weight holder into the hole next to the #9 index mark, and leave it there while performing the following steps.
— Progressively shorten the power arm l_2. To do this, hook the traction cord of the dynamometer in the holes next to the #10, #9, ..., #6 index marks (cf. Table 1, lower part), Measure F_2 in each case and record the respective values.

Results
see Table 1

Evaluation
Complete Table 1 with the concrete values for l_1 and l_2. (The distance between the index marks is 2 cm.) Subsequent to the calculation of the products $F \cdot l$, the following can be seen:

$$F_1 \cdot l_1 = F_2 \cdot l_2.$$

A one-sided lever is in equilibrium when the product of the conversely acting forces and their power arms are equal. In this context, the distance between the points of application of the forces and the fulcrum of the lever is termed the power arm.

Remarks
The lever is supported on a centre pivot so that the weights of the two lever halves are in balance (better: their torques). The students must be made aware of this fact in order* that they do not develop any false conceptions of a one-sided lever. It is advisable to cover the free (left) side of the lever with white paper, if desired, so that the image of a one-sided lever will be clear from the very beginning.
If the term torque can be used at this time, generalise the determined law to the momentum theorem:
On a one-sided lever the sum of all moments of rotation (torques) is equal to zero at equilibrium. For two forces, the following is true:

$$\vec{M} = \vec{F_1} \times \vec{l_1} + \vec{F_2} \times \vec{l_2} = 0.$$

where right and left torques have different signs:

$$\vec{F_1} \times \vec{l_1} = -\vec{F_2} \times \vec{l_2}.$$

For the case in which $\vec{F}$ and $\vec{l}$ enclose an angle of 90°,

$$|\vec{M}| = M = |\vec{F} \times \vec{l}| = F \cdot l,$$

However, the general case is
$M = F \cdot l \cdot \sin\alpha$ where $l \cdot \sin\alpha$ is also termed the effective length of the power arm or the effective lever length.
We consciously avoided the use of the terms load and load arm because load is not a physical quantity.

Table 1

Index No.	l_1 / cm	F_1 / N	$\frac{F_1 \cdot l_1}{N \cdot cm}$	Index No.	l_2 / cm	F_2 / N	$\frac{F_2 \cdot l_2}{N \cdot cm}$
10	20	1.95	39.0	10	20	1.95	39.0
8	16	1.95	31.2	10	20	1.56	31.2
6	12	1.95	23.4	10	20	1.18	23.6
4	8	1.95	15.6	10	20	0.78	15.6
2	4	1.95	7.8	10	20	0.40	8.0
9	18	0.98	17.6	10	20	0.88	17.6
9	18	0.98	17.6	9	18	0.98	17.6
9	18	0.98	17.6	8	16	1.11	17.8
9	18	0.98	17.6	7	14	1.26	17.6
9	18	0.98	17.6	6	12	1.48	17.8

Double-sided lever and more than two forces

Investigate under which conditions equilibrium exists on a double-sided lever if more than two forces are acting on it.

Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	2
Slotted weight, 10 g, black	02205.01	4
Slotted weight, 10 g, silver	02205.02	4
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Lever	03960.00	1
Pointer for demonstration lever	03963.00	1
White board pen, water soluble		

Set-up and procedure

— Place the axle on fixing magnet approximately in the middle of the demonstration board and slip the middle hole on the lever onto the axle.

— Using the white board pen, draw a vertical line downwards from the axle.

— Fix the pointer for demonstration lever onto the lever (in the following its tip lies directly on the vertical line when the lever is in equilibrium).

— Place one dynamometer onto the demonstration board. Measure the respective weights of the two weight holders which have been loaded differently. Record their weights.

— Hook the weight holder and the traction cord of the dynamometer at any arbitrary index marks on the lever. Move the dynamometer until the lever is horizontal and the traction cord of the dynamometer is perpendicular to the lever (Fig. 1).

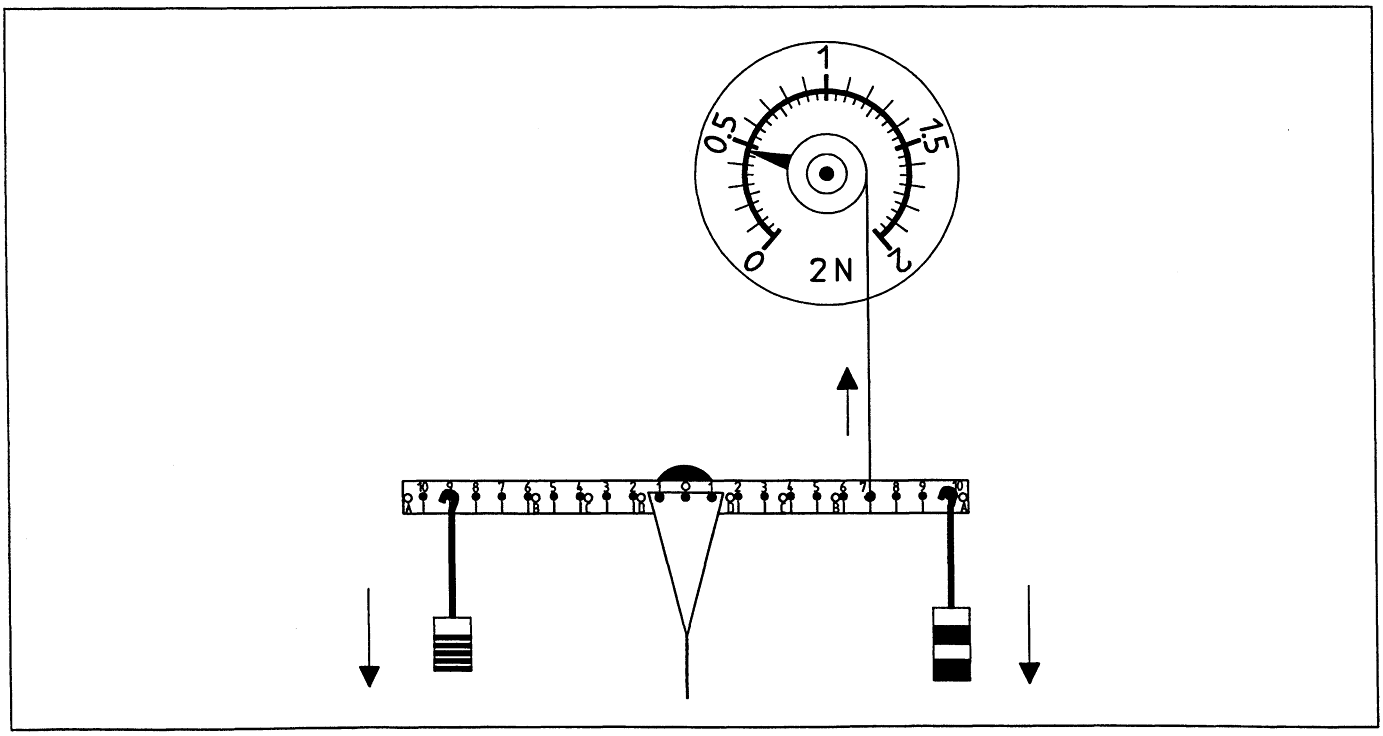
— Record the force which the dynamometer now indicates, as well as the weight of the loaded weight holders and their corresponding points of application (index mark numbers) in the upper part of Table 1. While doing so note which forces act clockwise and which, counterclockwise.

— Change the position of the clockwise-rotating weight holder (e.g. as given in Table 1, middle part).

— Change the position of the dynamometer and now allow the force to act on an index mark on the left-hand side. Adjust for equilibrium as before.

— Measure the force required to maintain equilibrium (now clockwise), and record it as well as the weights for the weight holders and the index mark numbers in middle section of Table 1.

Fig. 1



— Place the second dynamometer above the right side of the lever, change the position of the right-hand weight holder. After selecting an index mark, hook the dynamometer's traction cord into the respective hole on the lever.
— Move the second dynamometer until the lever is again horizontal and the traction cord is vertical to the lever.
— Record the forces indicated by the two dynamometers, those exerted by the weight holders, and the index numbers in the lower part of Table 1.

Results
The weights of the two weight holders with slotted weights:
$F_1 = 1.36$ N
$F_2 = 1.54$ N

Evaluation
After the individual index marks have been entered in Table 1 in correspondence with the power arms l, form the respective products $F \cdot l$. The results of the three partial experiments, which become obvious after the comparison of columns 4 and 8 in Table 1, are clearly presented in Table 2:
In accordance with these results and making allowances for the measuring accuracy, the following is true:
A double-sided lever on which more than two forces act is in equilibrium when the sum of the products of the forces acting in an counter-clockwise direction and their power arms is equal to the sum of the products of the forces acting in a clockwise direction and their respective power arms.

$$F_{1l} \cdot l_{1l} + F_{2l} \cdot l_{2l} + \ldots = F_{1r} \cdot l_{1r} + F_{2r} \cdot l_{2r} + \ldots$$

Remarks
The forces selected in the partial experiments can be greatly varied with the available experimental equipment. The measured data only represent sample data.
If the term torque can be used, the experimental result can be formulated more briefly:
A (double-sided) lever is in equilibrium when the sum of its moments of rotation (torques) are equal to zero.

$$\sum_{n=1}^{m} \vec{F}_n \times \vec{l}_n = 0.$$

Or:
A (double-sided) lever is in equilibrium when the sum of the clockwise torques are equal to the sum of the counterclockwise torques.
For the case in which, as in this experiment, $\vec{F}$ and $\vec{l}$ enclose an angle α of 90°,

$$|\vec{M}| = |\vec{F} \times \vec{l}| = F \cdot l \cdot \sin\alpha = F \cdot l,$$

Therefore, the equation formulated at the end of the evaluation is valid.

Table 2

Partial experiment	Sum of the products $F \cdot l$ for the Forces acting counter	Sum of the products $F \cdot l$ for the Forces acting clockwise
1	31.1 Ncm	30.8 Ncm
2	24.5 Ncm	24.4 Ncm
3	36.3 Ncm	36.2 Ncm

Table 1

Partial experiments		counter-clockwise				clockwise			
		Index mark no.	l / cm	F / N	$\frac{F \cdot l}{N \cdot cm}$	Index mark no.	l / cm	F / N	$\frac{F \cdot l}{N \cdot cm}$
1	Slotted weights	9	18	1.36	24.5	10	20	1.54	30.8
	Dynamometer	7	14	0.47	6.6	–	–	–	–
2	Slotted weights	9	18	1.36	24.5	6	12	1.54	18.5
	Dynamometer	–	–	–	–	4	8	0.74	5.9
3	Slotted weights	9	18	1.36	24.5	10	20	1.54	30.8
	Dynamometer	5	10	1.18	11.8	4	8	0.67	5.4

Investigate how large the load (supporting force) on two supporting pillars (supports) is, which, e.g., support a bridge on which there are vehicles.

Equipment

Demonstration board for physics	02150.00	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	2
Slotted weight, 10 g, black	02205.01	4
Slotted weight, 10 g, silver	02205.02	4
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Lever	03960.00	1
Thin wire		
White board pen, water soluble		

Set-up and procedure 1

— Draw a bridge on the demonstration board: two supporting pillars and a bridge element which is as long as the lever and horizontal.

— Affix wire loops, which are bent in such a manner that the lever that will subsequently be supported by them cannot tilt and slip off, to the holes at the #10 index marks at the ends of the lever.

— Place a dynamometer onto the demonstration board and measure the weight F_B of the lever (bridge element or beam); record its value.

— Also place the second dynamometer onto the demonstration board and hang the lever with the aid of the wire loops onto the traction cords. Move the dynamometers until the lever hangs horizontally in front of the drawing of the bridge element and the traction cords are perpendicular to it.

— Complete the drawing on the demonstration board by adding power arrows (vector arrows) (Fig. 1).

— Measure the supporting forces $F_{A,1}$ and $F_{A,2}$ and record their values.

Results 1

F_B = 1.23 N
$F_{A,1}$ = 0.61 N
F_B = 0.62 N

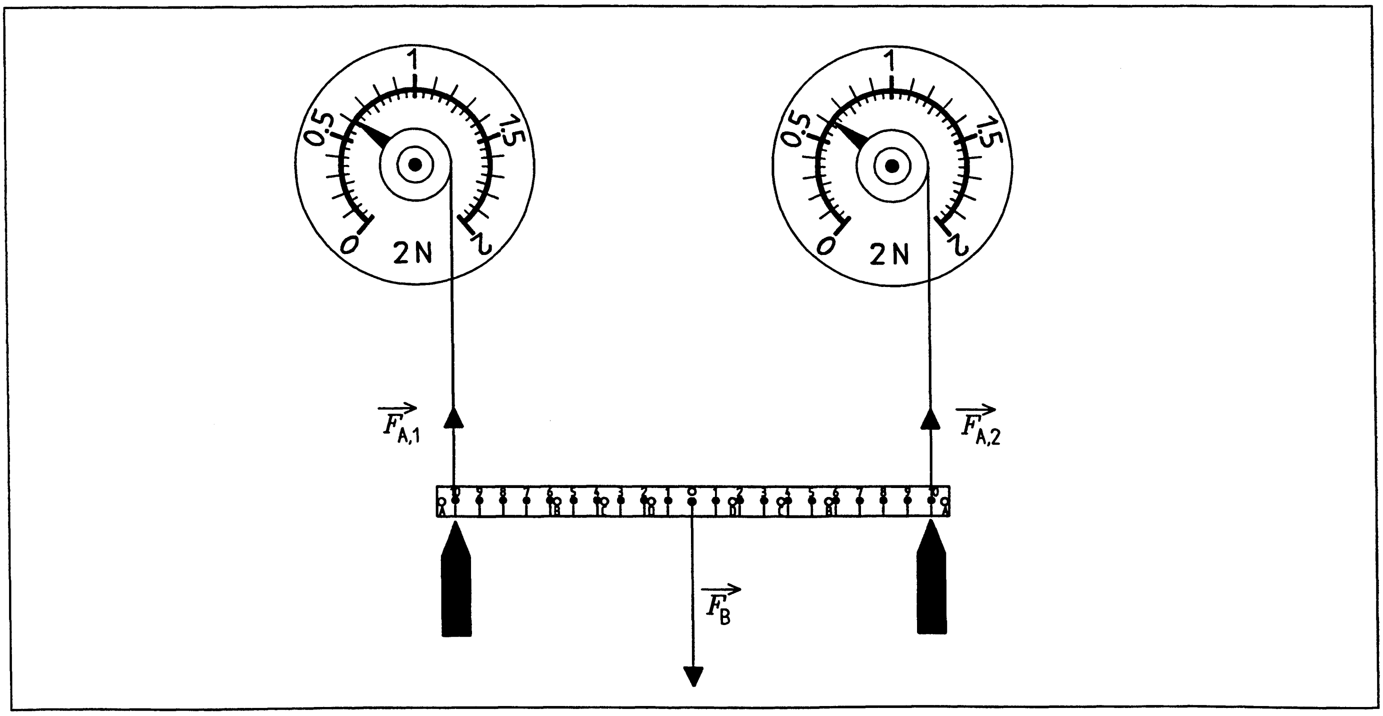
Evaluation 1

As the students can predict,

$$F_{A,1} + F_{A,2} = F_B ,$$

i.e., the sum of the supporting forces is equal to the weight of the bridge which bears no additional load to begin with.
Furthermore, the supporting forces are equal.

Fig. 1



Set-up and procedure 2

- Load one weight holder with slotted weights (3 x 50 g), and measure its weight with a dynamometer.
- Record F_1 and also F_B from Experiment 1.
- Hang the weight holder successively at different index marks. Shift the dynamometer in each case until the lever is horizontal and the traction cords are perpendicular to it. In each case measure the respective loading forces $F_{A,1}$ and $F_{A,2}$ and record them in Table 1. Also note the numbers of the index marks to which the respective F_1 is applied.

Results 2

$F_1 = 1.54$ N
$F_B = 1.23$ N

Table 1

$\vec{F}_1$ acts on index mark no.	$F_{A,1}$/N	$F_{A,2}$/N	$l_{1,1}$/cm	$l_{1,2}$/cm
7, right	0.85	1.93	36	4
4, right	1.07	1.72	28	12
5, left	1.77	1.02	10	30

Evaluation 2

To begin with – in accordance with the results of Experiment 1 – the students realise that $F_{A,1} + F_{A,2} = F_1 + F_B$.
The supporting forces can be calculated using the law of levers: The experimental set-up is considered to be a one-sided lever whose fulcrum coincides with the point of application of the force $F_{A,1}$ or $F_{A,2}$. As a consequence, the following is true (cf. Fig. 2):

$$F_{A,2} \cdot l_{A,2} = F_B \cdot l_B + F_1 \cdot l_{1,1}$$

or

$$F_{A,1} \cdot l_{A,1} = F_B \cdot l_B + F_1 \cdot l_{1,2}.$$

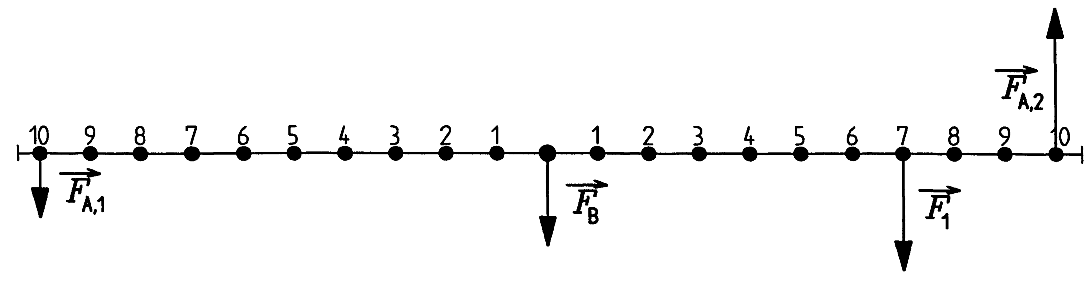
whereby $l_{1,2}$, for example, is the power arm of F_1 with respect to the point of application of $F_{A,2}$; l_B is the power arm of F_B, where F_B acts on the centre of gravity of the lever, therefore $l_B = 20$ cm.
In practice, the point of interest is the magnitude of the supporting forces under load. If one desires to calculate $F_{A,1}$, e.g., then the following is true:

$$F_{A,1} = \frac{F_B \cdot l_B + F_1 \cdot l_{1,2}}{l_{A,1}},$$

where

$F_{A,1} = (1.23 \text{ N} \cdot 20 \text{ cm} + 1.54 \text{ N} \cdot 4 \text{ cm})/40 \text{ cm}$
$= 30.8 \text{ N} \cdot \text{cm}/40 \text{ cm},$
$F_{A,1} = 0.77$ N.
This agrees with the measured value for $F_{A,1}$.

Fig. 2



Set-up and procedure 3

- Load the second weight holder with slotted weights (8 x 10 g + 1 x 50 g)) and measure its weight F_2 with a dynamometer.
- Record F_2 and also F_B and F_1 from Experiment 2.
- Hang the weight holder from appropriate index marks (if necessary use the dynamometer in the 4-N region). Perform the measurement of the supporting forces $F_{A,1}$ and $F_{A,2}$ as above.
- Record the supporting forces and the numbers of the index marks on which the respective $\vec{F}_1$ and $\vec{F}_2$ act.

Results 3

F_2 = 1.36 N, acts on index mark no. 2, right
F_1 = 1.54 N, acts on index mark no. 4, right
F_B = 1.23 N, acts on the centre of gravity of the beam/the bridge
$F_{A,1}$ = 1.55 N
$F_{A,2}$ = 2.60 N

Evaluation 3

Once again, making allowances for the range of measuring accuracy, the following result is obtained:

$$F_{A,1} + F_{A,2} = F_1 + F_2 + F_B.$$

The measured loading forces agree with those resulting from the following approach (cf. Fig. 3):

$$F_{A,1} \cdot l_{A,1} = F_1 \cdot l_{1,2} + F_2 \cdot l_{2,2} + F_B \cdot l_B$$

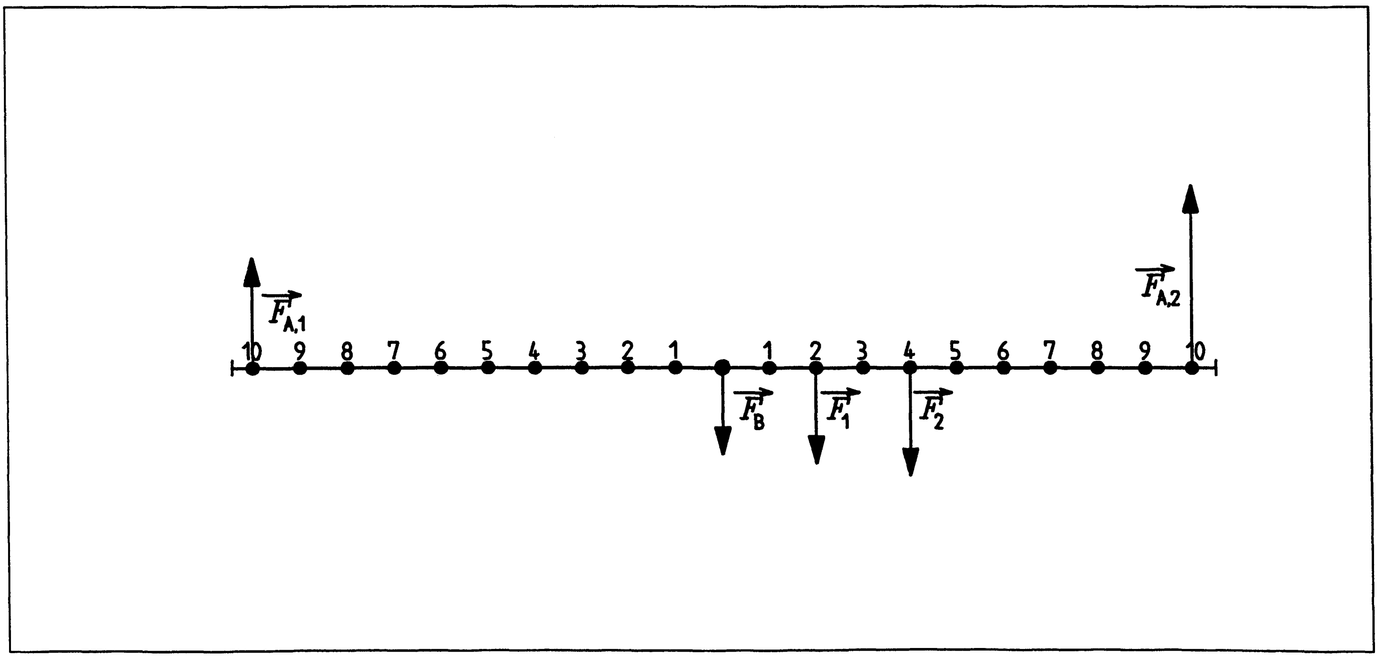
or

$$F_{A,2} \cdot l_{A,2} = F_1 \cdot l_{1,1} + F_2 \cdot l_{2,1} + F_B \cdot l_B$$

Remarks

If there is not enough time available, it is also sufficient, e.g. in Experiment 2, to determine only one value each for $F_{A,1}$ and $F_{A,2}$ or only to perform the Experiments 1 and 3 from which the most important aspects of supporting forces can be seen.
The considerations for the calculation of the supporting forces are in principle an application of the law of conservation of torque: a one-sided lever is in a state of equilibrium when the sum of the clockwise moments of rotation (the torques) is equal to the sum of the counterclockwise ones.
If there is no appropriate wire available to support the lever, it can also be supported with fish line in the "A" holes. In this case, lever arms of other lengths must be considered.

Fig. 3



Space for notes

Moment of rotation (torque)

Demonstrate that a body which can be rotated about an axis on which forces act eccentrically remains at rest if it is counterbalanced by the torques induced by the forces (equilibrium of torques).

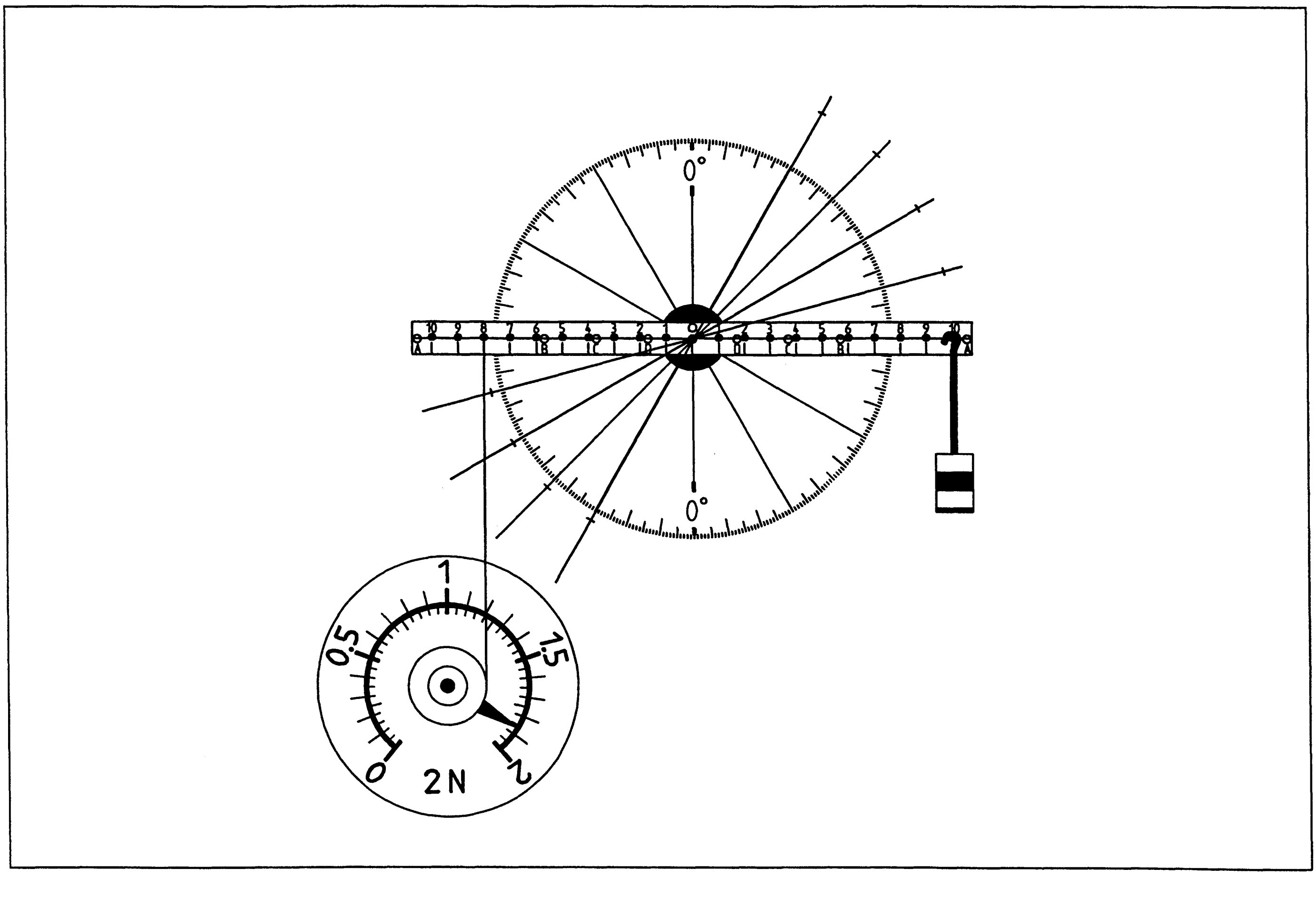
Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	2
Slotted weight, 10 g, black	02205.01	4
Slotted weight, 10 g, silver	02205.02	4
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Lever	03960.00	1
Protractor disk, magnet held	08270.09	1
White board pen, water soluble		
Triangle (set square)		

Set-up and procedure 1

- Place the protractor disk onto the demonstration board.
- Using the scale, draw lines through the centre of the disk which form angles of 15°, 30°, 45° and 60° with the horizontal line and which are approximately 22 cm long (seen from the centre) (cf. Fig. 1).
- Load one weight holder with slotted weights (3 x 50 g). Place a dynamometer onto the demonstration board and measure the weight F_1 of the loaded weight holder. Record F_1 in Table 1.
- Place the axle on fixing magnet onto the protractor disk in such a manner that the axle is located directly at the centre of the disk.
- Place the lever onto the axle such that the former can be rotated about its centre of gravity.
- Demonstrate that the lever can remain in any position (is in equilibrium).
- Hang the weight holder in the hole at the right #10 index mark and the traction cord of the dynamometer at the left #8 index mark. Shift the dynamometer until the lever is horizontal and the traction cord of the dynamometer is perpendicular to the lever (Fig. 1).
- Record the force F_2 indicated by the dynamometer.

Fig. 1



— Also note the length of the power arms $l_1 = l_{w1}$ and $l_2 = l_{w2}$.
— Rotate the lever by the intended intervals of 15°; do not change the index marks on which $\vec{F}_1$ and $\vec{F}_2$ act. In each case move the dynamometer such that its traction cord is perpendicular to the force arm when F_2 is to be measured. Record the respective forces F_2.
  Note: The angle between the force $\vec{F}_2$ and its power arm is exactly a right angle when the force indicated by the dynamometer is at its minimum.
— Remove all devices except for the protractor disk.
— With the scale on the lines which pass through the centre of the protractor disk mark the respective points of application of the forces $\vec{F}_1$. Using the triangle, drop a perpendicular to the horizontal line, thus determining the end point of the effective power arms $\vec{l}_{W1}$.
— Measure the effective power arms l_{w1} for F_1 for the angle α (angle between power arm and force); record the values in Table 1.

Results 1
See Table 1

Evaluation 1
From Table 1, it first follows that for a constant force F_1, the smaller the angle between the force and its power arm – and thus the smaller the (vertical) distance between the lines of action of $\vec{F}_1$ and the fulcrum of the body on which $\vec{F}_1$ acts – the smaller the products $F_1 \cdot l_{W1}$. This distance is termed the effective length l_w of the power arm l; the product $F_1 \cdot l_{W1}$, the moment of rotation (torque).

This is to be explained while the drawing on the demonstration board is being completed (cf. Fig. 2; explanation for the case where $\alpha = 30°$).
As a result of the torque produced by the force $\vec{F}_1$, the lever is turned clockwise, i.e. to the right. This torque acts against the one resulting from $\vec{F}_2$. If the body (the lever) is at rest, the moments of rotation (the torques) are equal. If the products $F_1 \cdot l_{W1}$ and $F_2 \cdot l_{W2} = F_2 \cdot l_2$, are formed, this fact will be confirmed (cf. columns 6 and 7 in Table 1).
A body which can be rotated about an axis and on which forces act eccentrically remains at rest if the moments of rotation produced by the forces counterbalance each other. It is then said that the body is in moment-of-rotation (torque) equilibrium.

Fig. 2

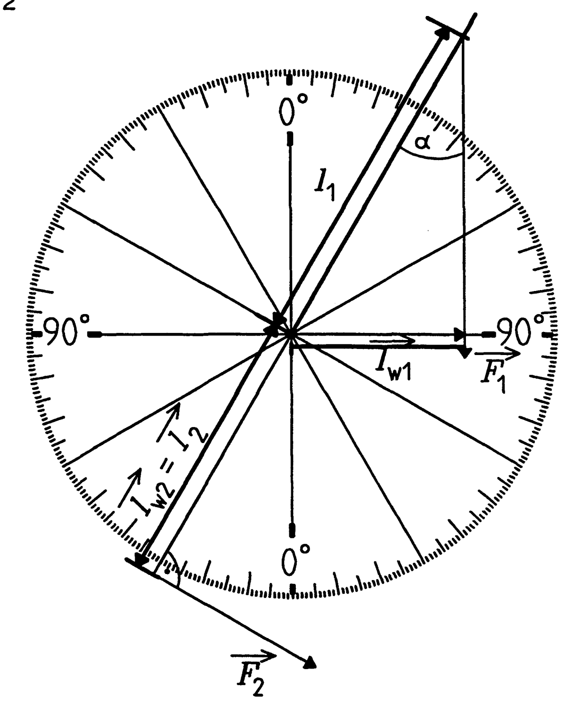


Table 1

α /1°	F_1 / N	F_2 / N	l_{W1} / cm	l_{W2} / cm	$\frac{F_1 \cdot l_{W1}}{N \cdot cm}$	$\frac{F_2 \cdot l_{W2}}{N \cdot cm}$
90	1.54	1.86	20.0	16.0	30.8	29.8
75	1.54	1.79	19.3	16.0	29.7	28.6
60	1.54	1.69	17.3	16.0	26.6	27.0
45	1.54	1.40	14.3	16.0	22.0	22.4
30	1.54	1.00	10.2	16.0	15.7	16.0

Set-up and procedure 2

— Place the axle and lever as in Experiment 1.
— Load the second weight holder (8 x 10 g + 1 x 50 g) and measure its weight F_2.
— Record F_2 and weight F_1 for the other loaded weight holder (1.54 N) in Table 2.
— Place both dynamometers onto the demonstration board.
— Hook the traction cords of the dynamometers and the two weight holders into appropriate holes (index marks) on the lever (cf. Fig. 3 and Table 2, column 2).
— Change the positions of the dynamometers until the lever is horizontal and the traction cords are perpendicular to it.
— Measure and record F_3 and F_4 as well as the respective power arms.

Results 2

See Table 3

Evaluation 2

The results of Experiment 1 also apply for more than two moments of rotation which act on a body.

Remarks

The special case where $\alpha = 90^\circ$ for all forces was chosen in Experiment 2 solely because it simplifies the determination of the power arms. If there is sufficient time available, one can, e.g., allow $\vec{F}_3$ and $\vec{F}_4$ to act on the lever at another angle or prepare the particularly involved case in which neither the lever is horizontal nor do the forces $\vec{F}_3$ and $\vec{F}_4$ act at a right angle to it.

The moment of rotation (torque) is a vectorial quantity; indeed, it is the cross product of force and vectorial power arm:

$$\vec{M} = \vec{F} \times \vec{l}.$$

For the values obtained here, the following is true:

$$|\vec{M}| = M = |\vec{F} \times \vec{l}| = F \cdot l \cdot \sin\alpha\ ,$$

where α is the angle enclosed by $\vec{F}$ and $\vec{l}$. $\vec{F}$ and $\vec{l}$ define a vector parallelogram, to which the vector $\vec{M}$ is perpendicular.

The evaluation of the experiments will be greatly facilitated if the students have appropriate knowledge of trigonometry. In that case, one can calculate l_w according to $l_W = l \cdot \sin\alpha$. The guide lines through the centre of the protractor disk should however also be drawn in this case because they facilitate the exact definition of the angle α.

Fig. 3

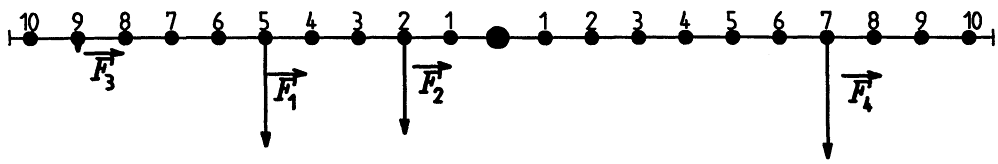


Table 2 (Sample measurements)

Force	Index mark no.	F / N	l / cm ($l = l_W$)	$\frac{F \cdot l}{\text{N} \cdot \text{cm}}$	$\frac{F \cdot l}{\text{N} \cdot \text{cm}}$
F_1	5, left	1.54	10	15.4	
F_2	2, left	1.37	4	5.5	} 24.3
F_3	9, left	0.19	18	3.4	
F_4	7, right	1.72	14	24.1	24.1

Space for notes

Beam balance

Demonstrate that the mass of bodies can be determined with a beam balance.

Equipment

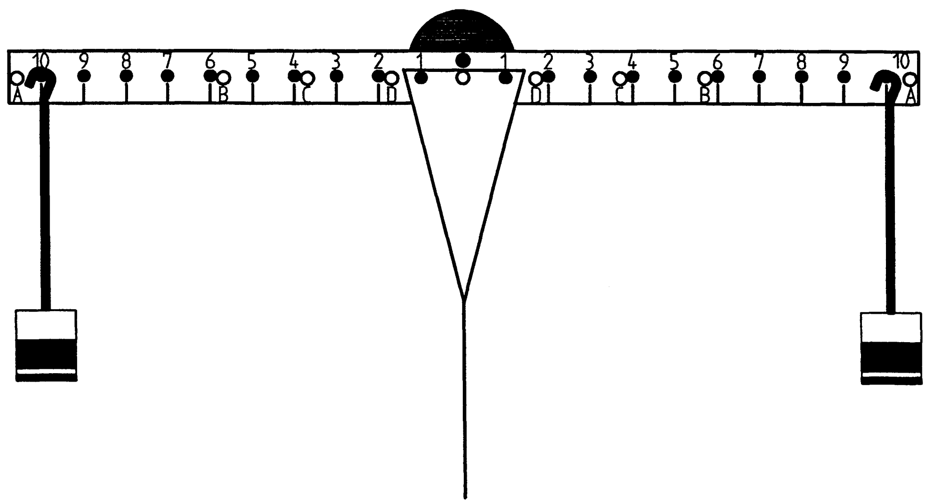
Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	2
Slotted weight, 10 g, silver	02205.02	2
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Balance pan	03951.00	2
Lever	03960.00	1
Pointer for demonstration lever	03963.00	1

Set of precision weights, 1...50 g (e.g. 44017.00)
White board pen, water soluble
Various objects

Set-up and procedure

— With the aid of the scale, draw a vertical line on the demonstration board
— Place the axle on fixing magnet onto the board in such a manner that the upper end of the line is covered and the axle lies directly on the line.
— Slip the lever onto the axle (the upper hole in the middle of the lever, so that the lever is in stable equilibrium) and attach the pointer (Fig. 1).
— Load the two weight holders with the same number of slotted weights (e.g. each with 1 x 10 g and 2 x 50 g). Hang them on the #10 index marks at the right and at the left ends.
— Hang one of the two weight holders on any arbitrary mark, and observe the lever. Determine where the second weight holder must be hung in order for the lever to again be horizontal.
— Record your observations under (1).
— Rehang the weight holders on the two #10 index marks. Remove one 10-g slotted weight, replace it, note your observation under (2).
— Remove the weight holders and hang the balance pans onto the lever. Pull the lever toward the end of the axle so that the balance pans are not in contact with the demonstration board (Fig. 2).
— Place various objects whose mass is unknown onto one of the balance pans; load the other balance pan with weights from the set of weights such that the balance is in equilibrium.
— Note the mass of the objects under (3).

Fig. 1



Results

(1) The lever is in equilibrium when the same bodies are hung at equal distances from the fulcrum.

(2) The lever is in equilibrium when the bodies which are hung at equal distances from the fulcrum have the same mass.

(3) Table

Body	Mass in g
Helical spring 3N/m	15
Movable pulley, *d* = 65 mm	12
Rod for pulley	15
Shaft	40

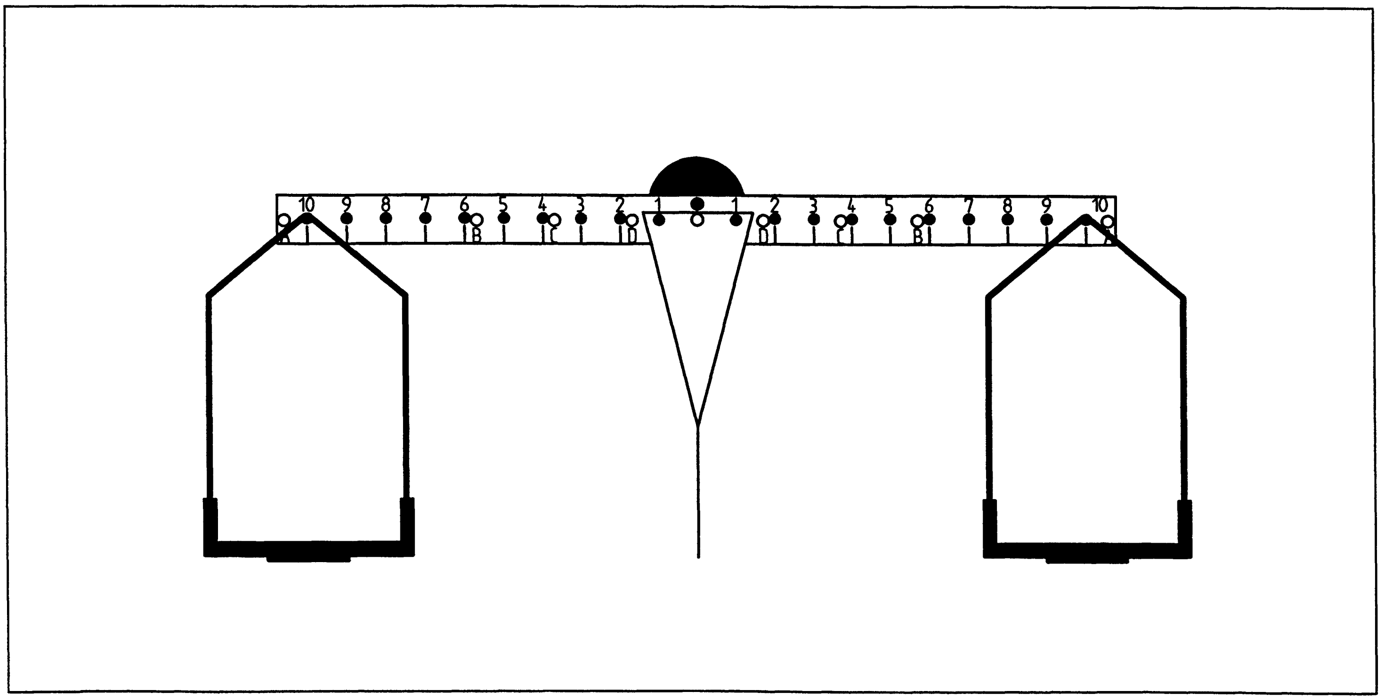
Evaluation

The double-sided, equal-armed lever (class 1 lever) is in equilibrium when the two acting forces are equal.
If, as in this experiment, the forces acting on it are the weights of objects, the weights of these bodies are equal, i.e. also their masses.
With the thus-characterised beam balance one can thus compare the masses of objects. As a consequence, it is possible to measure the masses of objects if one compares the unknown mass with the known mass of standardised mass pieces (weights).

Remarks

Selecting equal lever lengths (lengths of the power arms) for the beam balance has the advantage that the mass of an object can be determined without conversion or additional equipment. If unequal lever arms had been chosen, the balance would also be in equilibrium when the moments of rotation (torques) counterbalanced each other.
Unequal arms are selected in the sliding weight balance.
After concluding the described experiment, it is advisable to determine the mass of additional objects (measurement range is limited by the weight set) or to allow them be determined by the students.
Strictly speaking, during weighing, moments of rotation (torques) or – because the power arms are equally long – weights are compared. However, since the bodies are in the same place in the gravitational field of the earth, the weights are equal when the masses are equal.
The precondition required in order to be able to use the weighing procedure to determine the mass of bodies is thus gravitation, regardless of which heavenly body the object of unknown mass is located on. In a space ship circling the earth, it is impossible to weigh anything because of the compensation of the weight force (radial force) by the centrifugal force. This should be discussed with the students in a manner appropriate to their level of knowledge.

Fig. 2



Construct a model of a sliding weight balance and use it to determine the masses of objects.

Equipment

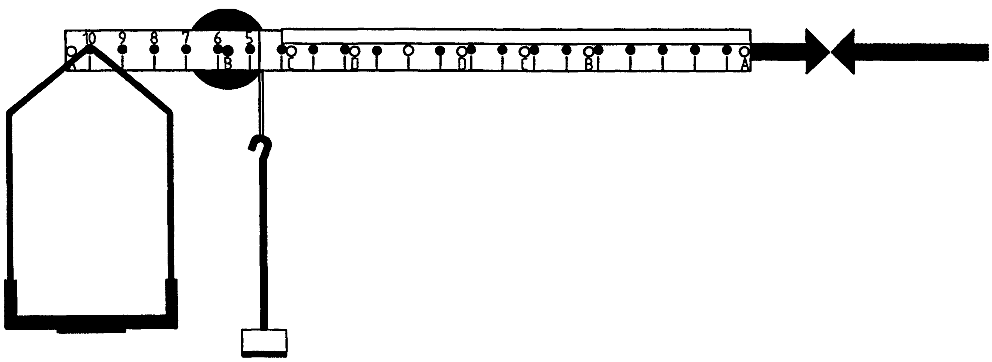
Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 1 out of	02204.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	4
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Weight, 150 g	11060.01	1
Balance pan	03951.00	1
Lever	03960.00	1
Fish line, 0.1 m of	02090.00	1

Set of precision weights, 1...50 g (e.g. 44017.00)
Various objects
Adhesive strips of paper, approx. 2 cm wide, approx. 40 cm long
Scissors
White board pen, water soluble

Set-up and procedure

— Paste the upper part of the lever over with an approximately 32 cm long piece of adhesive strip. From the remainder of paper strip cut out an arrow that is similar to the pointer for the demonstration board and paste it onto the back of the lever in such a manner that it extends a sufficient distance beyond the end of the lever (cf. Fig. 1).
— Make a loop of approximately 10 cm of fish line.
— Place the axle on fixing magnet onto the demonstration board and fix the lever onto the axle by means of the hole at point B.
— Hang the balance pan with the additional weight (150 g) at the #10 index mark on the left end. Hang the weight holder which is loaded with a 50-g slotted weight over the right arm of the lever with the aid of the loop, and move it until the lever remains in a horizontal position.

Fig. 1



- Mark the position with the pointer for demonstration board, which points at the arrow pasted to the lever when the balance is balanced out (cf. Fig. 1). Mark the position of the loaded weight holder, which functions as the sliding weight, on the paper (right side of lever). This determines the zero point of the scale for the sliding weight balance. Measure the distance l between the zero point and the fulcrum (from the middle of the axle) and record its value in Table 1.
- Hold the lever tightly, place a 10-g slotted weight onto the balance pan and move the sliding weight until the balance is in equilibrium, i.e. the two arrow heads are (as nearly as possible) exactly opposite one another. Mark the position on the scale where the sliding weight is now located. Measure the distance between this location and the fulcrum and record it.
- Increase the load of the balance pan up to 200 g in 10-g steps and proceed in the same manner as above for each step.
- Remove the slotted weights from the balance pan and now determine the weight of appropriate objects (measuring range 200 g) (cf. Table 2).

Results

Table 1

m / g	l / cm	m / g	l / cm
0	1.2	110	17.3
10	2.7	120	18.6
20	4.2	130	20.1
30	5.6	140	21.5
40	7.1	150	23.1
50	8.5	160	24.5
60	9.9	170	26.0
70	11.4	180	27.4
80	12.8	190	28.9
90	14.3	200	30.4
100	15.7		

Table 2

Object on the balance pan	Distance of the sliding weight from fulcrum l / cm	Position of the sliding weight on the scale in the range of	Mass of object m / g
Weight set	22.2	140 ... 150	141
Shaft without magnet	5.2	20 ... 30	27

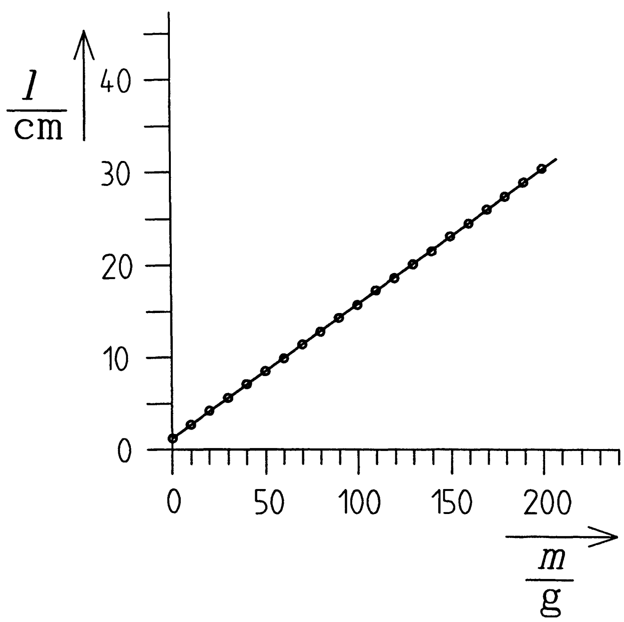
Evaluation

The double-sided, unequal-armed lever is in equilibrium when the clockwise and counterclockwise moments of rotation (torques) counterbalance each other.

In this experiment the acting forces are the weight of objects. Therefore, one can compare weights and thus masses with the device which is termed a sliding weight balance.

Through the comparison of the unknown mass of an object with the known mass of standardised mass pieces (weights) one can determine the unknown mass.

Fig. 2



On the sliding weight balance, equilibrium is achieved by changing the power arm of the sliding weight to the required length. Each length of the power arm corresponds to a specific mass of the body which is to be weighed. By proceeding according to this principle, one can calibrate the sliding weight balance.

Remarks

The calibration of the sliding weight balance in 10-g steps provides a great many gradations on its scale but is very time consuming. One could also proceed in larger steps (each 20 g or each 50 g in size) and subsequently perform a finer subdivision of the scale.

The most rational procedure would be to determine only the zero point and the end point (200 g) of the scale using measurements, and then to subdivide the scale linearly as finely as desired. This procedure is possible because a change in the counterclockwise force acting about the test object's weight (force) F_G (on the left arm of the balance) requires a proportional change in the length of the right power arm in order to restore the equilibrium of the balance:

$$\Delta F_G \sim \Delta l \text{ or } \Delta l \sim \Delta m \text{ (cf. Fig. 2).}$$

The counterclockwise moment increases proportionally to the mass of the test object (for a constant lever arm; the clockwise moment, proportionally to the length of the power arm (for constant force).

The determination of the masses of the test objects (cf. Table 2) is only as exact as the smallest-selected gradations of the scale. If the mass is to be determined as exactly as possible, one measures the length l of the lever arm required for equilibrium in each case and then calculates the mass using a known pair of measured values (e.g. 100 g $\hat{=}$ 15.7 cm). Otherwise, one can interpolate and obtain a good approximation of the mass. This latter method corresponds better to practical use of the balance.

Space for notes

Fixed pulley

Investigate the advantages which a fixed pulley has in the performance of mechanical work and which correlation exists between force and distance in this case.

Equipment

Demonstration board for physics	02150.00	1
Clamp on fixing magnet	02151.01	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 4 pieces	02204.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Pulley, moveable, with hook, *d* = 65 mm	02262.00	1
Rod for pulley	02263.00	1
Fish line, 2 m of	02090.00	1

Set-up and procedure

— Place the scale on the left side of the demonstration board. Position the clamp on fixing magnet with the rod and the pulley next to it at the upper edge (Fig. 1).

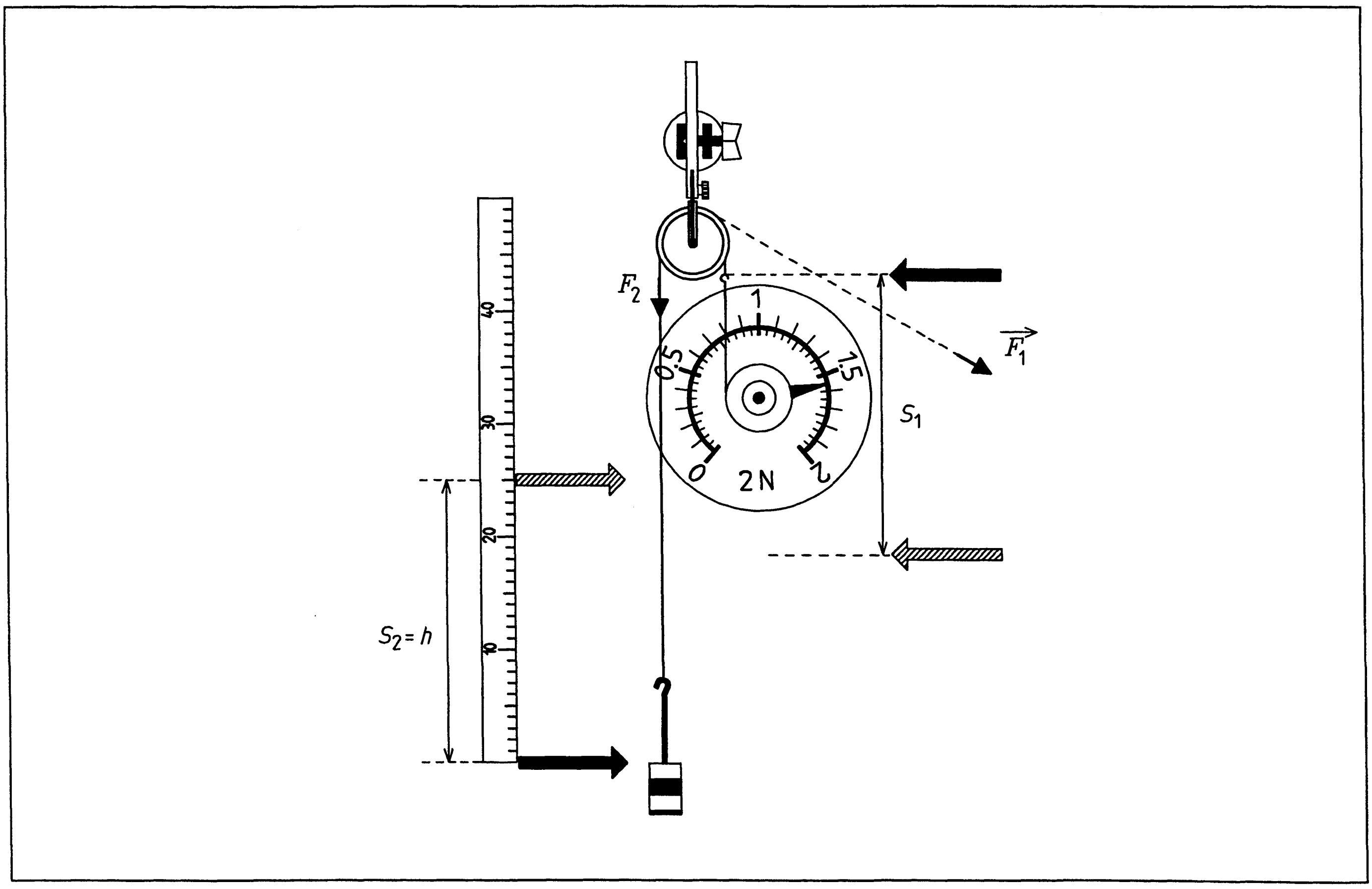
— Place the dynamometer below the pulley on the lower part of the demonstration board. Hang the weight holder which is loaded with three 50-g slotted weights onto the dynamometer. Measure $F_G = F_2$ for the loaded weight holder. Record the value for F_2 under (1).

— Move the dynamometer slowly and uniformly upwards (to the position corresponding to that in Fig. 1), and measure the force F required for the performance of lifting work. Record F (2).

— Lay a piece of cord (fish line) approximately 50 cm long with loops at its ends over the pulley. Hook the loaded weight holder and the traction cord into the loops (cf. Fig. 1). Record the force F_1 required at equilibrium of the pulley and make a statement concerning this under (3).

— Mark the position of the weight holder with an arrow and place the second arrow of the same colour somewhat higher, e.g. 25 cm.

Fig. 1



- Mark the position of the hook on the traction cord of the dynamometer with one of the different coloured arrows.
- Move the dynamometer uniformly downwards until the weight holder has been raised to the desired height $h = s_2$. Observe the dynamometer while doing so, and measure the force F_1 required for the performance of the lifting work. Record F_1 and F_2 as well as s_2 = h under (4).
- Mark the current position of the hook on the traction cord of the dynamometer with the fourth arrow (in Fig. 1 indicated by a hatched arrow). Measure the distance s_1 which was covered under expenditure of the force F_1 and record it under (4).
- Lower the weight holder and raise it again. However, this time, allow $\vec{F_1}$ to act in a different (cf. indicated direction in Fig. 1) and observe the dynamometer; if necessary, repeat the procedure several times. Record your observation under (5).

Results

(1) $F_G = F_2 = 1.54$ N
(2) $F = 1.54$ N
(3) $F_1 = 1.54$ N
Equilibrium exists when
$F_1 = 1.54$ N $= F_2$.
(4) $F_1 = 1.62$ N
$F_2 = 1.54$ N
$s_1 = 25$ cm
$s_2 = 25$ cm
(5) F_1 always has the same value regardless of the direction in which the force $\vec{F_1}$ acts.

Evaluation

A fixed pulley is in equilibrium when $F_1 = F_2$, i.e. when the tractive force is equal to the weight of the load. If lifting work is performed on a body without the use of a pulley, the tractive force is equal to the weight of the body. If the lifting work is performed with the aid of fixed pulley, a slightly larger force is required. This is due to the friction occurring on the axle of the moved pulley.

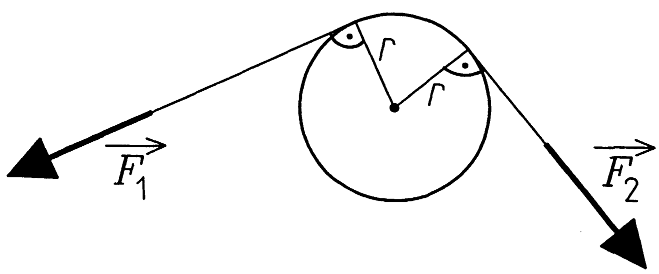
From the measured values the distances are obtained which were moved under the expenditure of the force F_1: $s_1 = s_2$,
$W_1 = F_1 \cdot s_1 = 1.62$ N $\cdot$ 0.25 m $= 0.40$ Nm,
for the work performed:
$W_2 = F_2 \cdot s_2 = 1.54$ N $\cdot$ 0.25 m $= 0.38$ Nm.

If the friction were to be reduced to the point where it would be negligible by using good bearings on the pulley, the following would be true: $W_1 = W_2$, i.e., the work expended is the same as the work performed, in this case the lifting work.
Therefore, no work can be avoided with a fixed pulley and the force which has to be expended is not reduced. Indeed, one must expend somewhat more work to overcome the friction which always occurs during movement. However, with a fixed pulley, one can change the direction of the forces. In practice, this is often a substantial advantage in the performance of mechanical work.

Remark

If the moment of rotation (torque) has been previously discussed, the equilibrium conditions for the fixed pulley $F_1 = F_2$ can be clearly illustrated with the aid of Fig. 2. The force arms for $\vec{F_1}$ and $\vec{F_2}$ are equal in length (equal to the contact radius of the lines of action which are tangential to the circle). Thus, F_1 and F_2 are also equally large.

Fig. 2



Investigate the advantages which a moveable pulley has in the performance of mechanical work and the correlation between forces and distances.

Equipment

Demonstration board for physics	02150.00	1
Clamp on fixing magnet	02151.01	1
Torsion dynamometer, 2 N/4 N	03069.03	2
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 4 out of	02204.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Pulley, moveable, with hook, $d = 65$ mm	02262.00	1
Pulley, moveable, with hook, $d = 40$ mm	03970.00	1
Rod for pulley	02263.00	1
Fish line, 2 m of	02090.00	1
White board pen, water soluble		

Set-up and procedure 1

— Place one dynamometer onto the demonstration board. Measure the weight of the weight holder which is loaded with three 50-g slotted weights including the pulley with $d = 65$ mm. Record F_G.

— Also place the second dynamometer onto the demonstration board. Prepare a piece of cord (fish line) which is approximately 60 cm long and has a loop on each end. Set-up the experiment according to Fig. 1.

— Measure the forces F_1 and record them.

— Compare F_1 with $F_G = F_2$.

Results 1

$F_G = 1.68\ \text{N} = F_2$

$F_1 = 0.84\ \text{N}$

$2F_1 = F_2$

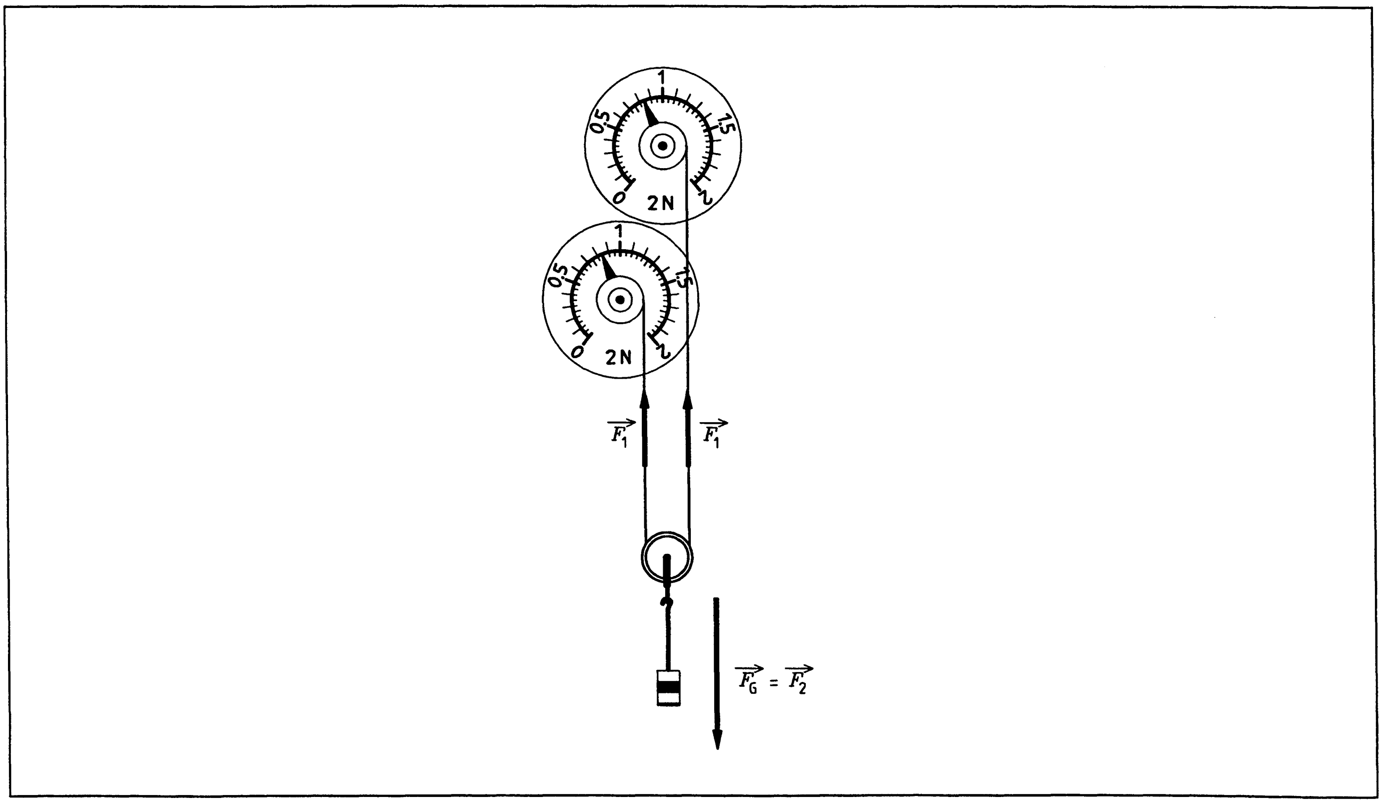
Evaluation 1

A moveable pulley is in equilibrium when the tractive force F_1 is half as large as the force F_2, the weight of the pulley and the load hanging on it:

$$F_1 = F_2/2.$$

This is because the force which is required to lift the load is divided between 2 pieces of rope.

Fig. 1



Set-up and procedure 2

- Place the hook on fixing magnet at the upper edge of the demonstration board and set up the experiment according to Fig. 2 in such a manner that the loaded weight holder extends somewhat beyond the lower edge of the board and that the dynamometer is at the lowest possible position.
- Mark the position of the weight holder's hook with the red arrow and the position of the hook on the dynamometer's traction cord with a blue one.
- Move the dynamometer slowly and uniformly in an upward, vertical direction as far as possible. While doing so, observe the deflection of the dynamometer and measure the force F_1 required to perform the lifting work. Record F_1.
- Mark the current position of the weight holder's hook and that of the dynamometer with the arrows of corresponding colours.
- Measure and record the distances covered s_1 (dynamometer) and s_2 (weight holder).

Results 2

Table 1

F_1 = 0.87 N
s_1 = 24 cm
s_2 = 12 cm

Evaluation 2

A comparison of the distances shows that $s_1 = 2s_2$. In the performance of work with the aid of a moveable pulley, the distance s_1 travelled by the point of application of the tractive force $\vec{F}_1$ is twice as large as the distance s_2 over which the load moves.

For mechanical work the following is true:
Work expended:
$W_{expn.} = W_1 = F_1 \cdot s_1 = 0.87\ \text{N} \cdot 0.24\ \text{m} = 0.21\ \text{Nm}$,
Work performed:
$W_{perf.} = W_2 = F_2 \cdot s_2 = 1.68\ \text{N} \cdot 0.12\ \text{m} = 0.20\ \text{Nm}$.

The work expended is slightly greater than that which is performed. This difference is due to the frictional force which occurs on the axle of the moveable pulley during movement.
As a good approximation, particularly in cases of low-friction mounting of the pulley:

$$W_1 = W_2$$

or

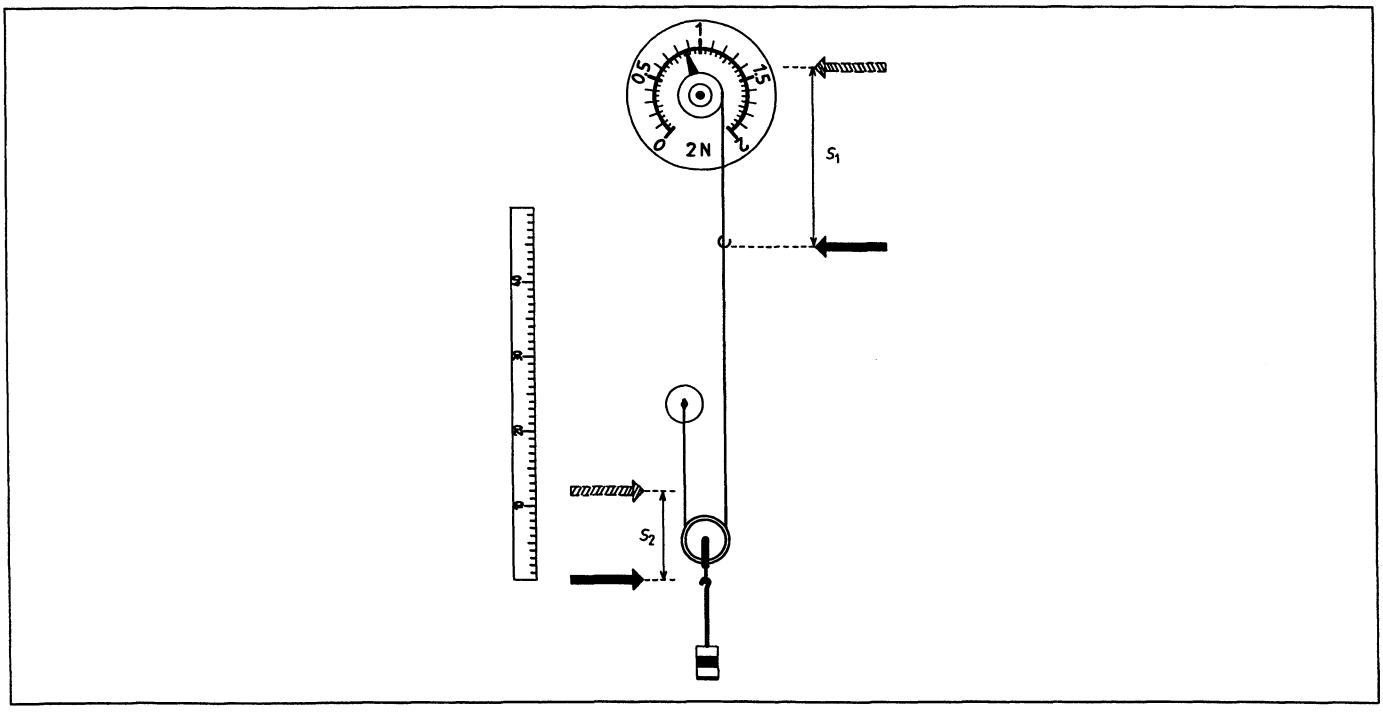
$$W_{expn.} = W_{perf.}$$

and

$$F_1 \cdot s_1 = F_2 \cdot s_2.$$

The use of a moveable pulley has the advantage that in the performance of mechanical work only half the force is required. On the other hand, one must accept a power distance which is twice as long. This is a special case of the golden rule of mechanics: performed work = expended work, which is valid as a good approximation for low-friction procedures.

Fig. 2



Set-up and performance 3

— Supplement the set-up of Experiment 2 as follows. Place the clamp on fixing magnet onto the demonstration board. Lay the cord over the pulley with d = 40 mm. Secure this pulley with the rod in the clamp (Fig. 3).
— Move the dynamometer uniformly downward several times. While doing so, measure the tractive force F_1. Record F_1.
— Also move the dynamometer in other directions. While doing so, pay attention to F_1. Record your observation.

Results 3

F_1 = 0.88 N
Observation: F_1 is always the same size, regardless of the direction in which one pulls.

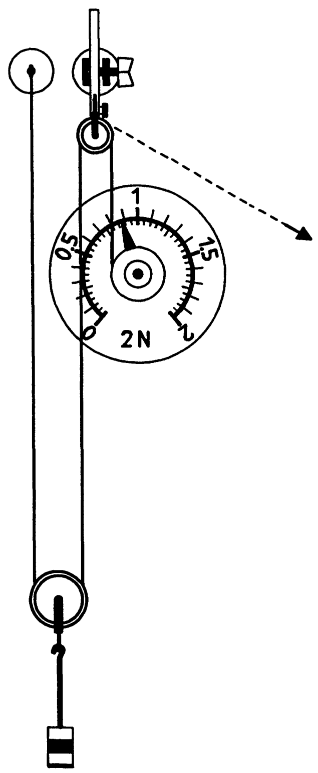
Evaluation 3

In this case, the frictional force of the second pulley must also be overcome when lifting work is performed. The work which is to be expended is thus somewhat larger than in Experiment 2. However, in practice the additional pulley provides the substantial advantage that one can perform lifting work with the moveable pulley more comfortably.

Remark

Experiment 3 is not required for working out the laws which are valid for a moveable pulley. But it increases the practical orientation of the investigations of the use of a moveable pulley.
It is also appropriate for transition to the treatment of the block and tackle.

Fig. 3



Moveable pulley

Space for notes

Investigate the advantages which a block and tackle provides in the performance of mechanical work and the correlations between forces and distances as a function of the number of supporting ropes.

Equipment

Demonstration board for physics	02150.00	1
Clamp on fixing magnet	02151.01	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 4 pieces	02204.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Rod for pulley	02263.00	1
Block and tackle with 4 pulleys	02265.00	1
White board pen, water soluble		

Set-up and procedure

— Load the weight holder with three 50-g slotted weights.

— Place the dynamometer onto the demonstration board and determine the weight $F_G = F_2$ for the loaded weight holder including a block with two pulleys. Record F_2.

— Place the clamp on fixing magnet with the rod for pulley in it onto the upper left-hand corner of the demonstration board.

— Thread the cord for the block and tackle around the pulleys and set-up the experiment according to Fig. 1. The length of the cord with two loops is approximately 160 cm.

— Measure the force F_1 required for equilibrium on the block and tackle and record it under (2).

— Compare F_1 with F_2 (2).

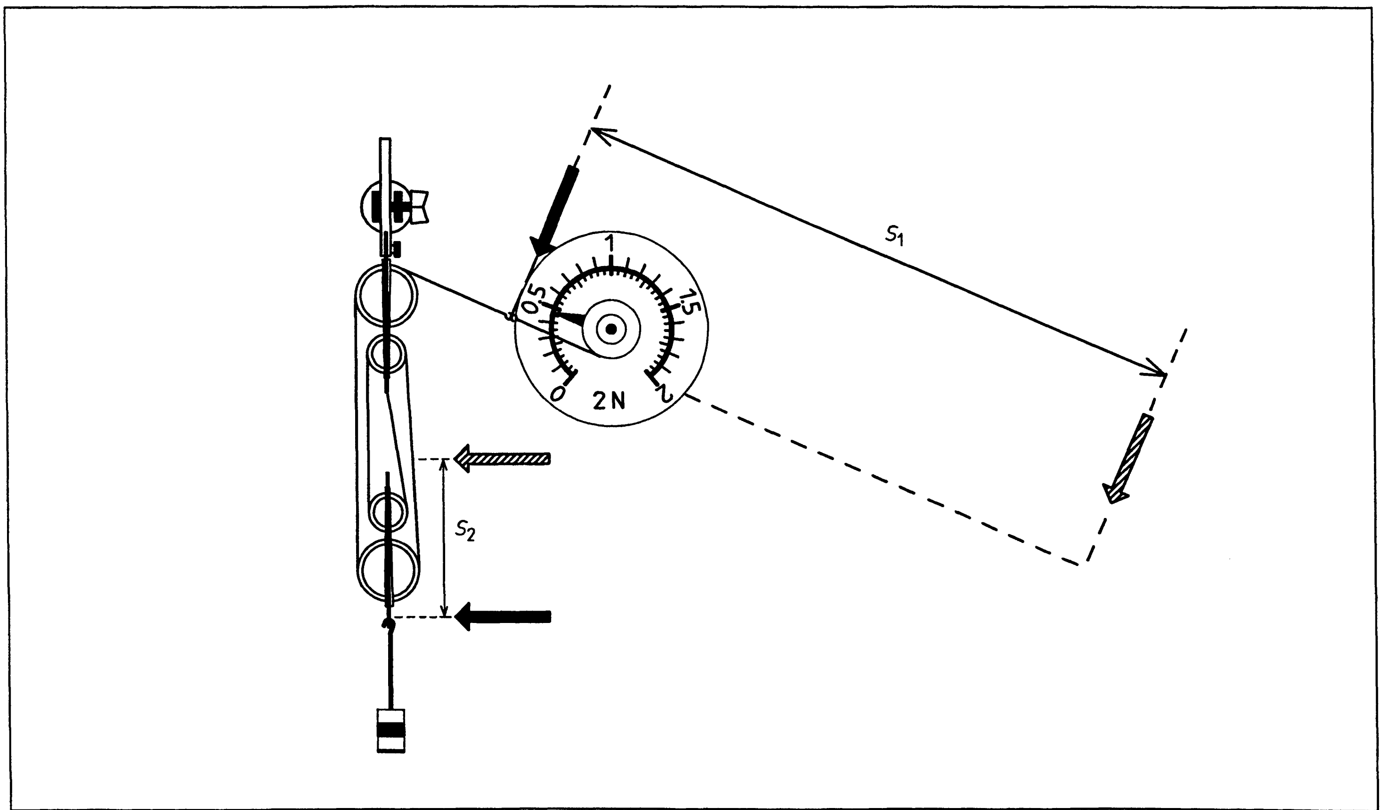
— Mark the position of the weight holder (highest point on the hook) and of the dynamometer (hook on the traction cord – cf. Fig. 1) with arrows of different colours.

— Move the dynamometer slowly and uniformly in the direction of the lower right-hand corner of the demonstration board. Measure force F_1 and record it under (3).

— Mark the current position of the weight holder's hook and the hook on the traction cord with arrows of colour corresponding to the above use.

— Mark the distance (lifting height) s_2 for the load and the force distance s_1 on the demonstration board. Measure s_1 and s_2 and record them under (3)

Fig. 1



— Raise the dynamometer repeatedly, and while doing so perform lifting work with differently oriented forces $\vec{F}_1$. Note your observation under (4).

Results

(1) $F_G = F_2 = 1.78$ N
(2) $F_1 = 0.44$ N
$F_1 = F_2/4$
(3) $F_1 = 0.47$ N
$s_1 = 60$ cm
$s_2 = 15$ cm
(4) F_1 always has the same value, i.e. 0.47 N.

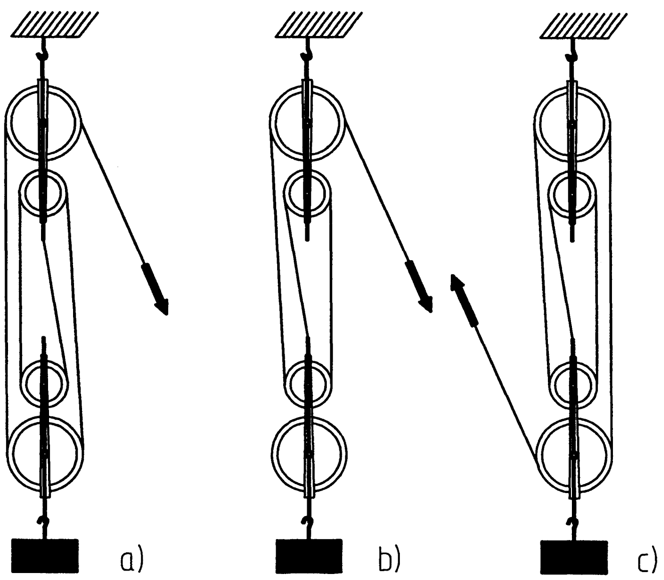
Evaluation

A block and tackle with 4 pulleys is in equilibrium when the tractive force is equal to 1/4 of the load's weight: $F_1 = F_2/4$.
This due to the fact that the weight of the load is divided among 4 load-bearing segments of rope since the block and tackle has 2 moveable pulleys. For one moveable pulley, the equilibrium condition was found to be $F_1 = F_2/2$. For the use of three moveable pulleys $F_1 = F_2/6$ would be valid; for n moveable pulleys, $F_1 = F_2/2n$. The important factor on a block and tackle is the number of load-bearing segments of rope!
For the performance of mechanical work (in this experiment lifting work) F_1 must be greater than that required for equilibrium because during movement the tractive force F_1 must also compensate the frictional forces which always occur on the bearings of the pulleys.
In addition, the measurements show that $s_1 = 4s_2$.
For work, the following is true:
expended work $= F_1 \cdot s_1 = 28$ N · cm $= W_1$,
performed work $= F_2 \cdot s_2 = 27$ N · cm $= W_2$.
The expended work W_1 is somewhat larger than the performed work W_2.
If the frictional force can be neglected in comparison to $F_2 = F_G$, the following is true:
$F_1 \cdot s_1 = (F_2/4) \cdot (4s_2) = F_2 \cdot s_2$ oder $W_1 = W_2$.
In summary, the following is true for a block and tackle with 2 moveable pulleys. On the one hand, the required tractive force is equal to 1/4 of the load's weight. On the other hand, the force distance is 4 times as large as the load distance (the lifting height). One cannot avoid work with a block and tackle, but it does make the performance of mechanical work substantially easier, especially since it makes no difference in which direction the tractive force acts.

Remarks

The fact that the force which is expended on a block and tackle is not a function of the number of pulleys, but rather that the number of load-bearing rope segments is the determining factor, must be made extremely clear. Fig. 2 indicates how this aspect can be further illustrated – also experimentally if necessary. For the same block and tackle F_1 can be $F_1 = F_2/4$ or $F_1 = F_2/3$ or $F_1 = F_2/5$ depending on whether path of the rope is selected according to case a), b), or c).
If the block and tackle with 6 pulleys (Order no.: 02264.00) is available, it can additionally be demonstrated that for three moveable pulleys with 6 load-bearing rope segments $F_1 = F_2/6$ is valid.

Fig. 2



Investigate the conditions under which equilibrium exists on a wheel and axle.
In addition, investigate the correlation between the forces , the distances and the radii on the wheel and axle.

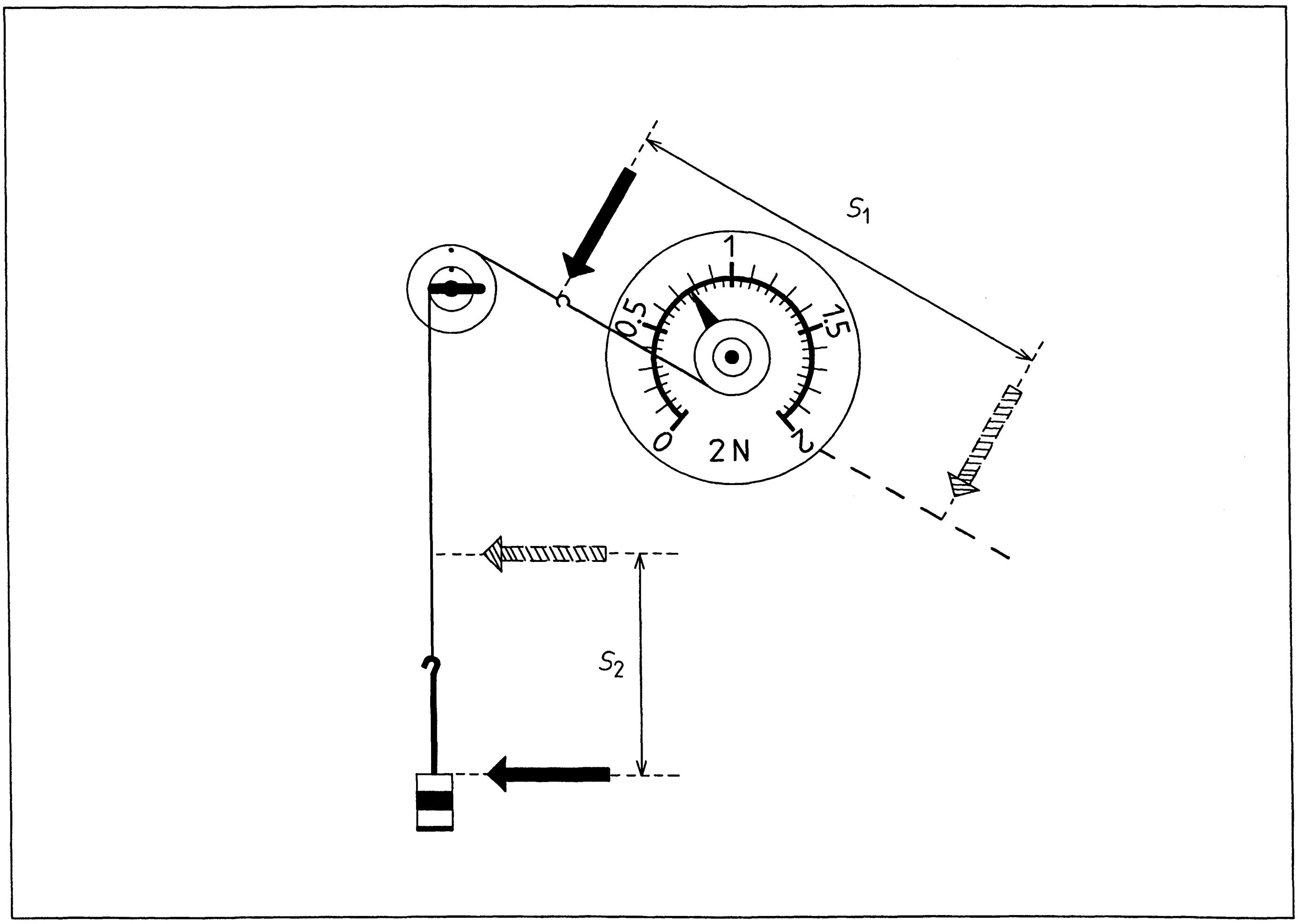
Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	1
Torsion dynamometer, 2 N/4 N	03069.03	1
Scale for demonstration board	02153.00	1
Pointers for demonstration board, 4 pieces	02204.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	2
Wheel and axle	02360.00	1
Fish line, 1 m of	02090.00	1
White board pen, water soluble		

Set-up and procedure

— Provide each of two pieces of cord (fish line), which are approximately 50 cm and 40 cm long, with a loop and thread them onto the circumference of the large and small wheels (pulleys), respectively, of the wheel and axle (step pulley).
— Place the axle on fixing magnet onto the upper part of the demonstration board.
— Screw the axle out of the threaded hole , place the two belt pulleys onto it and screw the shaft tight again. Using the crank, subsequently connect the two belt pulleys to form a rigid wheel and axle.
— Load the weight holder with three 50-g slotted weights.
— Place the dynamometer onto the demonstration board and measure the weight $F_G = F_2$ for the loaded weight holder. Record F_2 under (1).

Fig. 1



— Setup the experiment according to Fig. 1. Hang the weight holder on the line which is wrapped around the small wheel (pulley).
— Measure force F_1, which is required to maintain the equilibrium of the wheel and axle. Record F_1 and F_2 under (2).
— Measure (or give the students the values for)the power arms r_1 and r_2 for F_1 and F_2. Also record them.
— Mark the position of the weight holder and of the hook on the dynamometer's traction cord with different coloured arrows.
— Move the dynamometer slowly and uniformly diagonally downward (e.g. a distance of 45 cm). While doing so, observe the deflection of the dynamometer and record the required tractive force F_1 as well as $F_G = F_2$ under (3).
— Mark the current position of the weight holder and of the hook with arrows of colours corresponding to their previous use.
— Draw in and measure the distance s_2 (lifting height) of the load as well as the power distance s_1 on the demonstration board. Note the measured values.
— Record the measured values under (3).

Results

(1) $F_G = F_2 = 1.54$ N

(2) Table 1

Small wheel	Large wheel
$r_2 = 3.5$ cm	$r_1 = 7.0$ cm
$F_2 = 1.54$ N	$F_1 = 0.77$ N
$F_2 \cdot r_2 = 5.4$ Ncm	$F_1 \cdot r_1 = 5.4$ Ncm

(3) Table 2

Small wheel	Large wheel
$r_2 = 3.5$ cm	$r_1 = 7.0$ cm
$F_2 = 1.54$ N	$F_1 = 0.80$ N
$s_2 = 15.0$ cm	$s_1 = 30.5$ cm
$F_2 \cdot s_2 = 23$ Ncm	$F_1 \cdot s_1 = 24$ Ncm

Evaluation

The wheel and axle is in equilibrium when the clockwise and counterclockwise torques are equal (cf. Table 1, last line).

$$F_1 \cdot r_1 = F_2 \cdot r_2 \ .$$

or if the forces act conversely to the radii:

$$\frac{r_1}{r_2} = \frac{F_2}{F_1} \ .$$

If one performs mechanical work with the aid of the wheel and axle, the force which must be exerted is less than the force which must be overcome in accordance with the ratio of the radii to each other. In the experiment this is the weight of the load on which lifting work must be performed.

From (3) it follows that the expended work $F_1 \cdot s_1$ is slightly greater than the work performed $F_2 \cdot s_2$. In practice this is due to the friction which always occurs in the bushings of the pulley.

If it is possible to reduce the friction to a level where it can be neglected, the following is true:

$$W_{exp.} = F_1 \cdot s_1 = F_2 \cdot s_2 = W_{perf.}$$

As with other force transforming devices, no work can be avoided with the wheel and axle. However, in practice the performance of work can be substantially facilitated with the wheel and axle.

Remarks

In the literature one also finds the terms step pulley or step wheel for the wheel and axle. In earlier times, the use of the wheel and axle could be frequently observed in everyday life: at the village well, on ferries and on winches of various types, chains or ropes were wound around a shaft, which was activated with a hand wheel (crank) of much larger diameter than that of the shaft (axle).

In the treatment of the wheel and axle, one must not forget that the students have learning difficulties as a result of the use of the same units for torque and work (Nm or Ncm). This should be considered in planning the lesson.

Toothed-gearing

Demonstrate the construction and mode of operation of toothed-wheel gearing using a simple gear train.

Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	2
Gear-wheel, 20 teeth	02350.13	1
Gear-wheel, 40 teeth	02351.03	1
Crank from wheel and axle	02360.00	1
White board pen, water soluble		

Set-up and procedure

— Screw the axles out of their threaded holes, place a gear-wheel on each of them and retighten the axles. Place the fixing magnets onto the demonstration board in such a manner that the teeth of the gear-wheels mesh with each other slightly. The index marks (dots) on the gear-wheels should be aligned vertically (Fig. 1).

— Attach the crank to the large gear-wheel and turn it through a single revolution. In the process, pay attention to the movement of the two index marks as well as to the direction of rotation of the wheels.

— Draw arrows indicating the direction of rotation on the demonstration board with the white board pen (Fig. 1).

— Formulate a statement expressing your observations.

— Attach the crank to the small gear-wheel and turn it through several revolutions – also with the opposite rotational sense.

— Record your observations.

Results

The small gear wheel makes twice as many revolutions as the large one in the same period of time. The directions of rotation of the wheels are opposite.

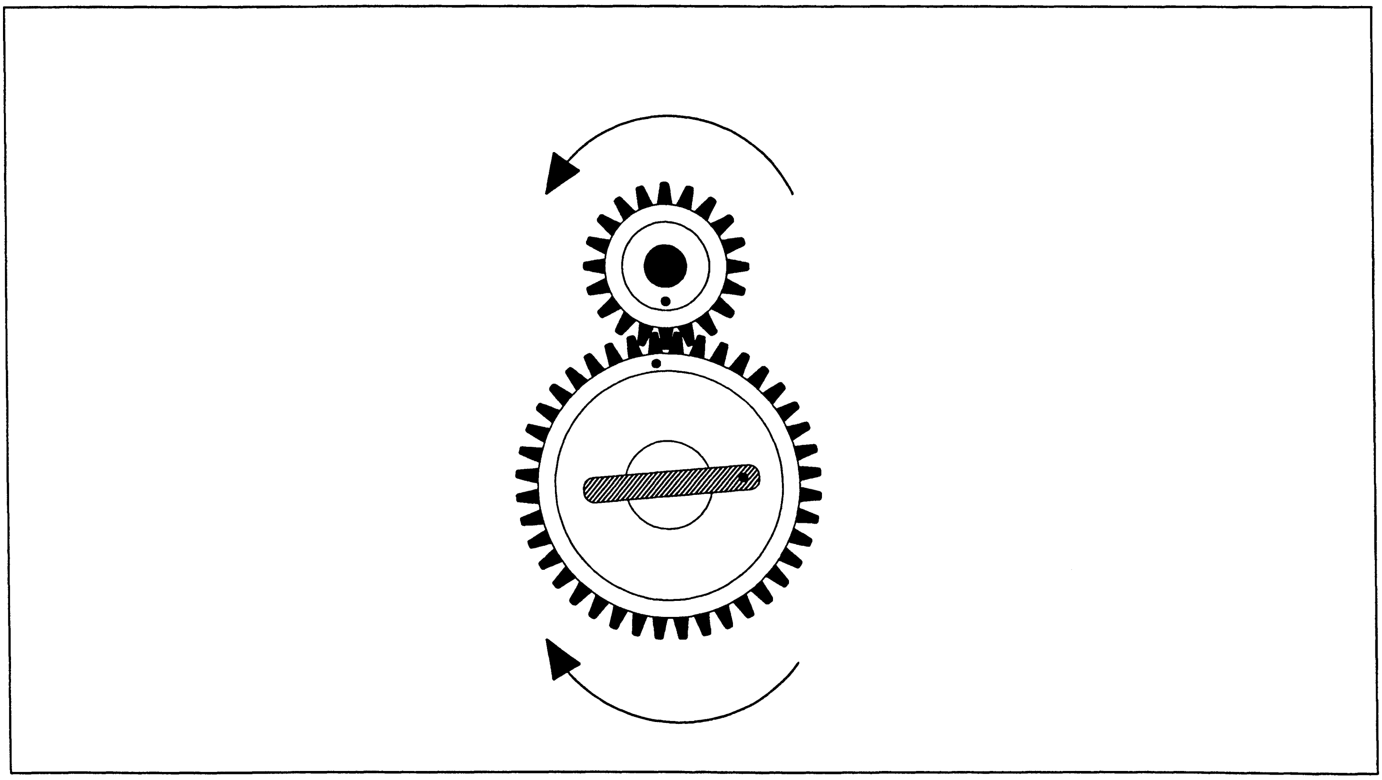
If the large gear-wheel is turned by the small one, the large one makes half as many revolutions in the same period of time and always in the opposite rotational direction.

Evaluation

Toothed-wheel gearing serves the transmission of force as well as the transmission and transformation of rotational movements.

The transmission of force occurs without slippage due to the form-fitted meshing of the gear-wheels. whereby the divisions *T* of the gear wheels (distance between one point of a tooth to the same point on the adjacent tooth) must be equal.

Fig. 1



Toothed-gearing

For the transmission of force the gear ratio i is decisive: $i = n_{tr} : n_g$
where
n_{tr} = Rotational velocity of the driving gear-wheel
and
n_g = Rotational velocity of the driven gear-wheel.

If N is the number of rotations of a gear-wheel during the time t, the following is true:

$$n = N / t$$

and thus

$$i = N_{tr} / N_g.$$

The circumferential velocities of the geared wheels must be the same as a result of the lack of slippage:

$$v_{tr} = v_g.$$

If Z_{tr} and Z_g are the number of teeth, it follows that:

$$Z_{tr} \cdot T \cdot n_{tr} = Z_g \cdot T \cdot n_g,$$

$$n_{tr} / n_g = Z_g / Z_{tr} = i.$$

The transmission ratio can thus be determined as the quotient of the number of teeth of the driven and of the driving gear-wheels.
In the experiment conducted here $Z_1 = 40$ and $Z_2 = 20$. Thus the ratio $Z_1 : Z_2 = 2 : 1$. As a consequence, the small wheel turns twice as fast as the large one regardless of whether it drives or is driven. In this experiment the transmission ratio first has a value of 1/2 and then 2/1.

Remarks
The torque which can be transmitted by toothed-wheel gearing is substantially greater than that which can be transmitted by a belt drive due to the positive interlocking of the gears.
Technical applications of toothed-wheel gearing, which should generally be known to the students, are, e.g., the transmission in cars or the gear drive in mechanical clockworks.

Demonstrate the construction and mode of operation of a belt drive using a single-stage belt drive.

Equipment

Demonstration board for physics	02150.00	1
Axle on fixing magnet	02151.02	2
Wheel and axle	02360.00	1
Rubber band, 1 out of	03920.00	(1)
White board pen, water soluble		

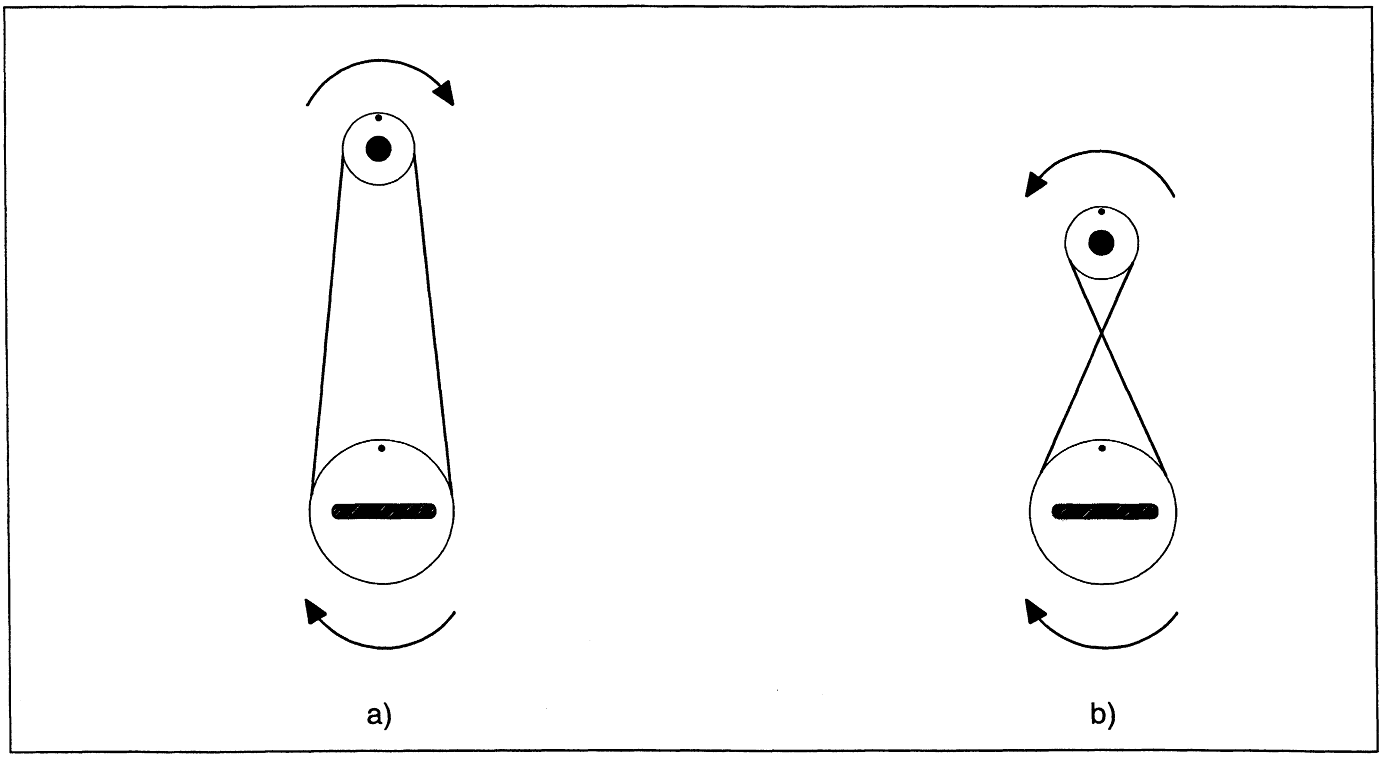
Set-up and procedure

— Screw the axles out of their respective threaded holes. Slip one of the pulleys from the wheel and axle onto each of them and screw the axles on tight again.
— Place the fixing magnets onto the demonstration board in such a manner that the rubber band which is placed into the groove of the pulleys has sufficient tension and still allows the pulleys to rotate easily. The index marks (dots) on the pulleys should be in a position which is easy to remember (Fig. 1a).
— Affix the crank to the large pulley and then turn the pulley through one revolution. While doing so, pay attention to the movement of the two index marks as well as to the rotational directions of the pulleys.
— Draw arrows on the board with the white board pen to indicate the rotational directions (Fig. 1).
— Make a statement describing your observations (1).
— Affix the crank to the small pulley and also rotate the crank in different directions.
— Note your observations.
— Remove the rubber band and replace it in a crossed configuration (Fig. 1b). To achieve this move one of the axles on fixing magnet slightly until the band is sufficiently taut and the pulleys can still be easily turned.
— Perform this belt-drive experiment in a manner analogous to that described above.
— Record your observations (2).

Results

(1) The small pulley rotates twice as fast as the large one. The rotational directions of the two belt pulleys are the same.
If the larger belt pulley is driven by the small one, the rotational directions are also the same and the driven large belt pulley turns half as fast as the small one.
(2) With a crossed belt, the same is true for the rotational speed as without the belt being crossed. However, the rotational directions of the two belt pulleys are now opposite.

Fig. 1



Evaluation

Belt drives serve to transmit force, and the transmission and conversion of rotational movements. In the process, the belt transmits torque from one belt pulley to the other.

The transmission of force occurs through frictional contact between the belts and the running faces of the belt pulleys. For the transmission of force the transmission ratio *i* is decisive:

$$i = n_{tr} : n_g$$

where

n_{tr} = Rotational speed of the driving belt pulley and

n_g = Rotational speed of the driven belt pulley.

If *N* is the number of revolutions of a belt pulley during the time *t*, then the following is valid:

$$n = N/t$$

and therefore

$$i = N_{tr}/N_g.$$

For a non-slip belt drive, the circumferential velocity of the two belt pulleys is equal:

$$v_{tr} = v_g$$

As a consequence, the following is true:

$$\pi \cdot d_{tr} \cdot n_{tr} = \pi \cdot d_g \cdot n_g \; (d = \text{diameter}).$$

It follows that:

$$n_{tr}/n_g = d_g/d_{tr} = i.$$

The transmission ratio can thus be determined from the diameter of the diameters of the driven and the driving belt pulleys.

In the experiment performed, d_1 = 70 mm and d_2 = 35 mm; thus the ratio $d_1{:}d_2$ = 2:1. As a consequence, the small belt pulley rotates twice as fast as the large one, regardless of whether it drives or is driven.

The transmission ratio first has a value of 1/2 and then 2/1 in the experiment.

Remarks

The transmission ratios 2/1 = 2 and 1 /2 = 0.5 were not exactly achieved in the experiment for the following reason:

The running surfaces for the drive belts are approximately 0.5 mm deep, thus the ratios of the diameters are approximately 2.03 and 0.49, respectively.

The two belt drive mechanisms demonstrated in this experiment are termed open and crossed belt drives.

Belt drives have the advantage that rotational movement can be transmitted between two shafts which are relatively far apart. Compared to the interlocking gear drive, they have the disadvantage that in the transmission of large torques slippage can occur.

The students are familiar with technical applications of belt drives from photographs of older factories as well as from the belt-driven crankshaft generator/alternator in cars.

Determine the physical quantities on which the oscillation period of a thread pendulum depends.

Equipment

Demonstration board for physics	02150.00	1
Clamp on fixing magnet	02151.01	1
Hook on fixing magnet	02151.03	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver	02206.02	1
Holding pin	03949.00	1
Protractor disk, magnet held	08270.09	1
Fish line, 0.5 m of	02090.00	1
Stopwatch		
White board pen, water soluble		
Triangle		

Set-up

— Place the protractor disk near the top left-hand corner of the demonstration board.
— Secure the holding pin in the clamp, and place the clamp in the centre of the protractor disk.
— Place the hook on fixing magnet to the right of the protractor disk.
— Thread a piece of cord (fish line) which is approximately 90 cm long through the hole in the holding pin (Fig. 2).
— Tie loops in the ends of the cord and complete the experimental set-up according to Fig. 1. To begin with, the weight holder is loaded with a 50-g slotted weight.
— Move the hook on fixing magnet until the pendulum length *l* is, e.g., 50 cm (Fig. 2). (The centre of gravity of the pendulum bob is approximately at the lever of the upper edge of the 50-g slotted weight.)
Note: In order to be able to position the centre of gravity of the pendulum at an exact distance *l* from the pivot point without large parallax errors, use both the scale and a triangle.
— Draw a vertical line downward from the pivot point of the pendulum, and mark the position of the pendulum's centre of gravity on it (Fig. 1).

Fig. 1

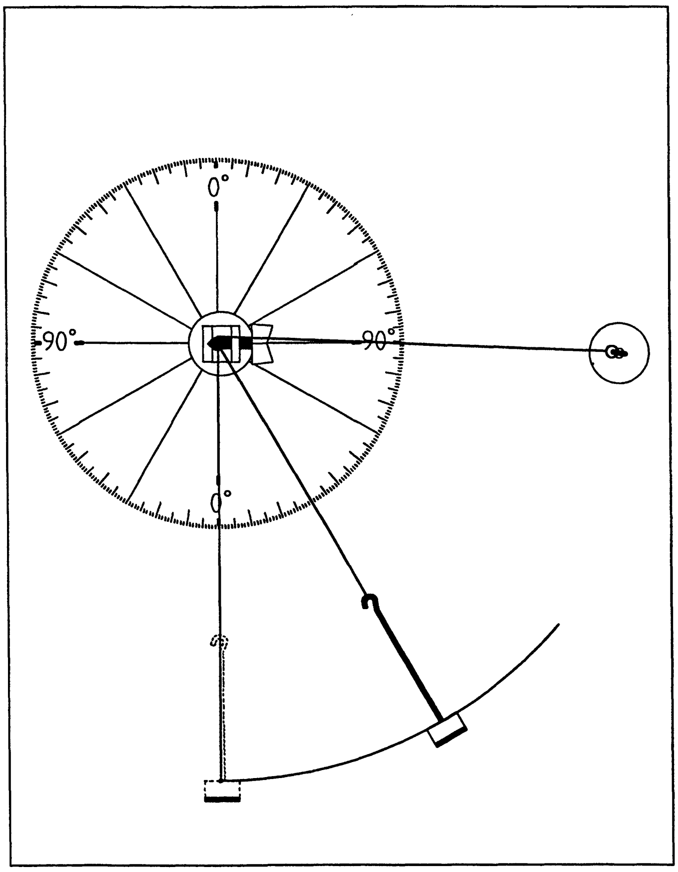
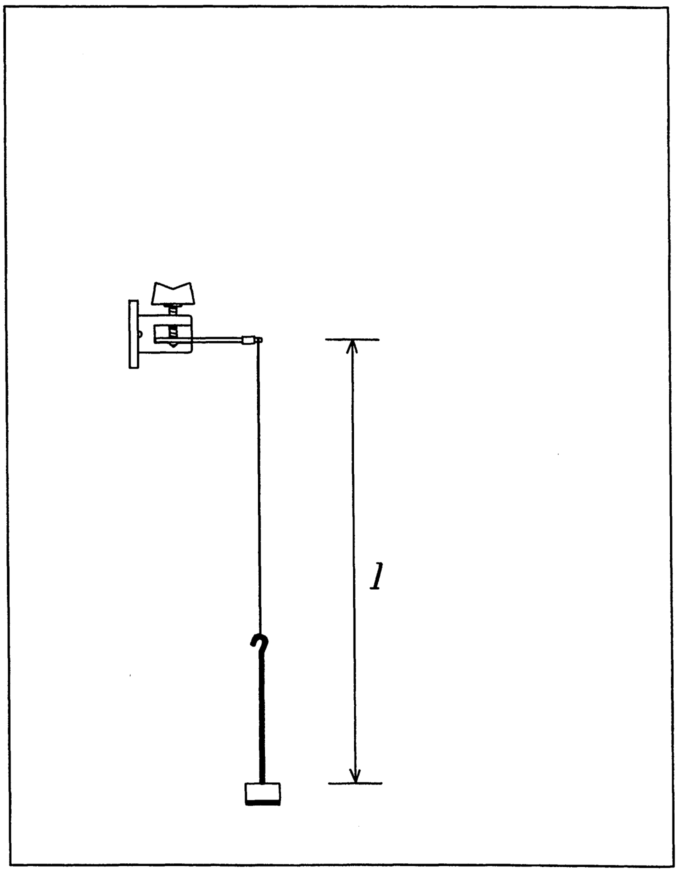


Fig. 2



Procedure 1

— Displace the pendulum (e.g. by approximately 30°) and release it. Measure the time 10 *T* required for 10 complete, sequential oscillations with the stopwatch. Record the mean for 10 *T* in Table 1.

— Displace the pendulum by a lesser distance (e.g. approximately 20°) and proceed as above.

— Perform the measurement for a small displacement (e.g. approximately 10°).

— Finally, displace the pendulum several times by $\alpha \approx 90°$ and in each case measure *T* for a single oscillation (due to the relatively large damping). Record the mean for *T*.

Results 1

Table 1

Displacement α /1°	10 *T* / s	*T* / s
30	14.5	1.45
20	14.4	1.44
10	14.2	1.42
90	–	1.80

Evaluation 1

The oscillation period of the thread pendulum is nearly independent of the size of the displacement α (or the amplitude of the oscillation). However, this is only true for relatively small values of the angle α.

Procedure 2

— Using an otherwise-unchanged experimental set-up, place a second 50-g slotted weight onto the weight holder; *m* is now 110 g.

— As above measure the time required for 10 complete oscillations at approximately the same, but small, amplitude several times and record the values for 10 *T* and *T* in Table 2.

— Complete Table 2 by adding the values for 10 *T* and *T* which were determined in Experiment 1 (*m* = 60 g).

Results 2

Table 2

m / g	10 *T* / s	*T* / s
60	14.4	1.44
110	14.3	1.43

Evaluation 2

The oscillation period of the thread pendulum is independent of the mass of the pendulum bob.

Procedure 3

— For otherwise unchanged experiment set-up, progressively shorten the length of the pendulum, e.g., by moving the hook on fixing magnet to the right (while doing so adhere to the information given in the note under Set-up).

— After adjusting the pendulum to the respective desired length, measure the time for 10 *T* and the mean value of *T* in each case. Record these values and those determined in the previous experiments for *l* = 50 cm in Table 3.

Results 3

Table 3

l / cm	10 *T* / s	*T* / s	T^2 / s^2	$\frac{T^2/l}{s^2/cm}$
50	14.4	1.44	2.07	0.0414
40	13.0	1.30	1.69	0.0422
30	11.2	1.22	1.25	0.0417
20	9.0	0.90	0.18	0.0405

Evaluation 3

The oscillation period *T* of a thread pendulum is a function of the pendulum length *l*. The smaller *l* is, the smaller *T* is.

But there is no proportionality between *T* and *l*. If one forms the quotients of T^2/l (cf. Table 3, last column), the result is a constant, whose value in this experiment is approximately 0.041 s^2/cm = 4.1 s^2/m.

Therefore, T^2/l = constant: This is equivalent to $T^2 \sim l$.

Draw the students' attention to the fact that the constant has a value which is approximately $4\pi^2/g$ = 4.02 s^2/m, where *g* is the acceleration of gravity.

It follows that:

$T^2/l = 4\pi^2/g$. After rearranging, this results in the following:

$$T = 2\,\pi \cdot \sqrt{l/g}\,.$$

Inform the students that this equation can also be obtained from theoretical considerations.

Remarks

The holding pin offers several advantages for mounting the thread pendulum. It guarantees sufficient clearance between the board and the pendulum if the plane of the board and the oscillation plane are not exactly parallel. It also allows exact definition of the pivotal point and simple alteration of the pendulum's length.

It is important for the accuracy of the measured values in Experiment 3 that the distance between the centre of gravity of the loaded weight holder and the suspension point is used as the pendulum length. One can demonstrate to the students that the centre of gravity has the described position by laying the shaft of the loaded weight holder on a loop of fish line and balancing it. Loading it with an additional slotted weight indeed shifts the centre of gravity, but only to a small extent.

The equation $T = 2\pi \cdot \sqrt{l/g}$ can serve as the basis for an additional experiment: construction of a "second pendulum".

In accordance with $l = T^2 g/4\pi^2$, the following is obtained for $T = 2 \cdot$ s: l = 99.4 cm. A pendulum having this length requires exactly one second for half an oscillation.

The thread pendulum is also termed gravity pendulum because it can only oscillate within a gravitational field.

The equation $T = 2\pi \cdot \sqrt{l/g}$ is only valid for small angles of displacement α, namely for $\alpha < 5°$. For this reason the amplitudes selected in Experiments 2 and 3 should also be kept small.

The students should additionally realise that the equation for the oscillation period can be used to determine the acceleration of gravity at the location of measurement:

$$g = 4\pi^2 l / T^2.$$

From the experimentally determined values, the following is obtained: $g = 4\pi^2 \cdot (0.041\ \mathrm{s^2/cm})^{-1}$ = 9.63 m/s^2 with a deviation of 1.8% from 9.81 m/s^2.

Space for notes

Spring pendulum

Determine the physical quantities on which the oscillation period of a spring pendulum depends.

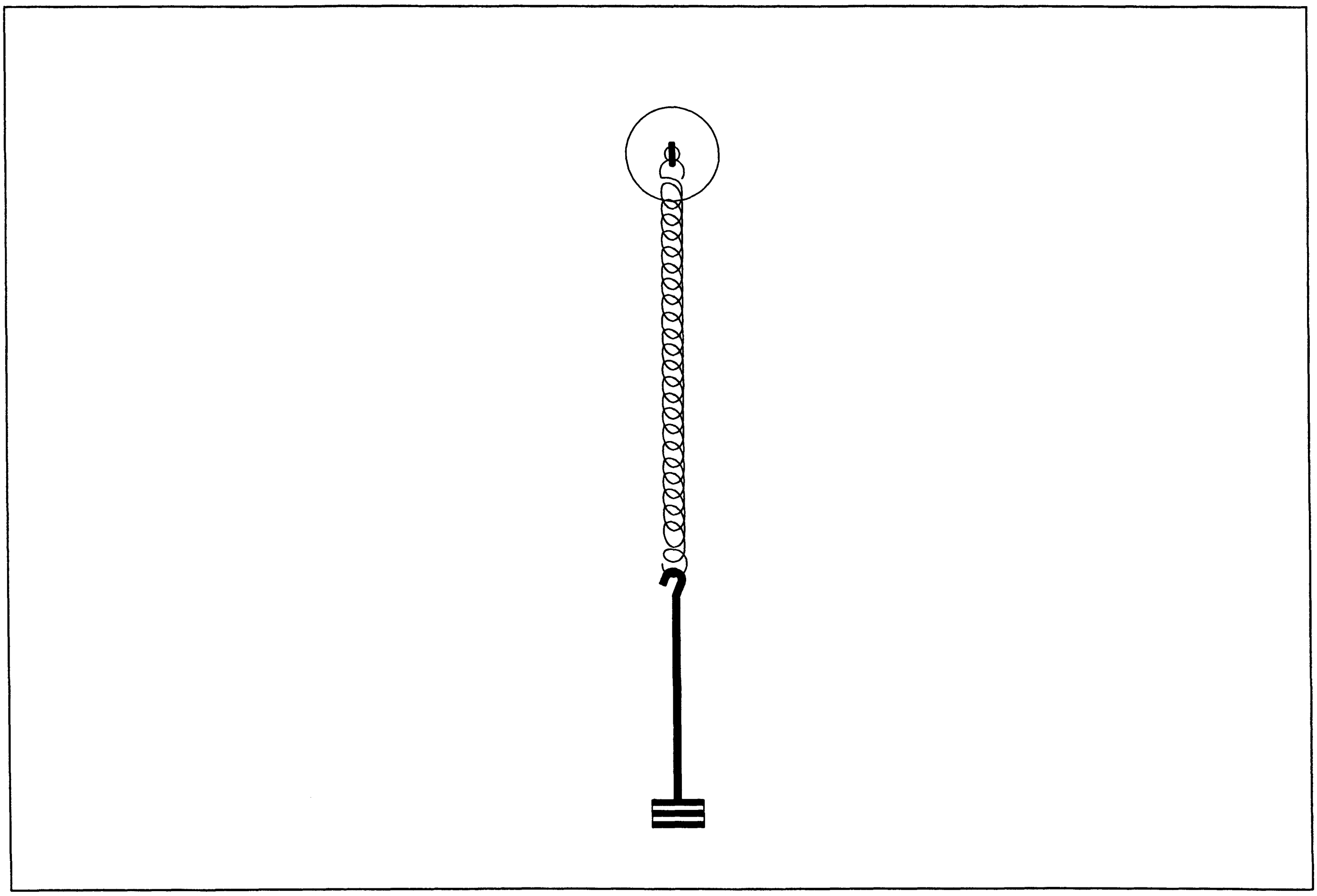
Equipment

Demonstration board for physics	02150.00	1
Hook on fixing magnet	02151.03	1
Helical spring, 3 N/m	02220.00	1
Helical spring, 20 N/m	02222.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver	02205.02	2
Slotted weight, 50 g, black	02206.01	2
Slotted weight, 50 g, silver	02206.02	2
Stopwatch		
White board pen, water soluble		

Set-up and procedure

- Place the hook on fixing magnet onto the demonstration board.
- Hang the helical spring with 20 N/m on the hook.
- Load the weight holder with four 10-g slotted weights (Fig. 1).
- Pull the weight holder several centimetres downward and release it. Measure the time 10 T required for 10 complete oscillations, and record the measured value for 10 T in Table 1.
  Note: For extremely rapid oscillations it is advisable to measure 20 or 30 T and to use the value obtained to determine 10 T for the subsequent calculations.
- Progressively load the weight holder in 50 g steps and determine the respective values for 10 T. Record them in Table 2.
- Hang the 3-N/m helical spring on the hook in place of the 20-N/m one.
- Load the weight holder with on e 10-g slotted weight, and initiate oscillation: Measure 10 T and record the value in Table 2.
- Increase the load on the weight holder in steps of 20 g each; proceed in the same manner as above.

Fig. 1



Results

Table 1 (20-N/m helical spring)

m / g	10 T / s	T / s	T^2 / s^2
50	3.3	0,.3	0.109
100	4.6	0.46	0.212
150	5.6	0.56	0.314
200	6.5	0.65	0.422
250	7.1	0.71	0.504

Table 2 (3-N/m helical spring)

m / g	10 T / s	T / s	T^2 / s^2
20	5.8	0.58	0.336
40	7.8	0.78	0.608
60	9.4	0.94	0.884
80	10.7	1.07	1.145
100	11.8	1.18	1.392
120	13.0	1.30	1.690
140	13.9	1.39	1.932

Evaluation

To begin with, calculate the values for T and T^2 and record them in Tables 1 and 2. In Fig. 2 the square of the oscillation period is plotted against the mass m. In both cases there is a linear correlation. Especially for the softer spring (3 N/m) on can clearly see that the straight line does not pass through the origin. The reason for this is that the mass of the springs compared to the mass of the variously loaded weight holder is not small enough to be neglected (cf. in particular the smallest m values with the mass of the spring).

Masses m_F of the helical springs:
20 N/m: $m_F = 5.7$g
3 N/m: $m_F = 15.8$ g

In order to obtain a proportional correlation (i.e. a plot in which the straight lines pass through the ordinate, see Fig. 3), the mass of the oscillating system must be corrected. To do so, add one-third of the mass of the spring to the mass m in each case. The corrected mass which is to be used in the further calculations is thus

$$m_k = m + \Delta m$$

where
$\Delta m = 1.9$ g for the 20-N/m spring and
$\Delta m = 5.3$ g for the 3-N/m spring.

Fig. 2

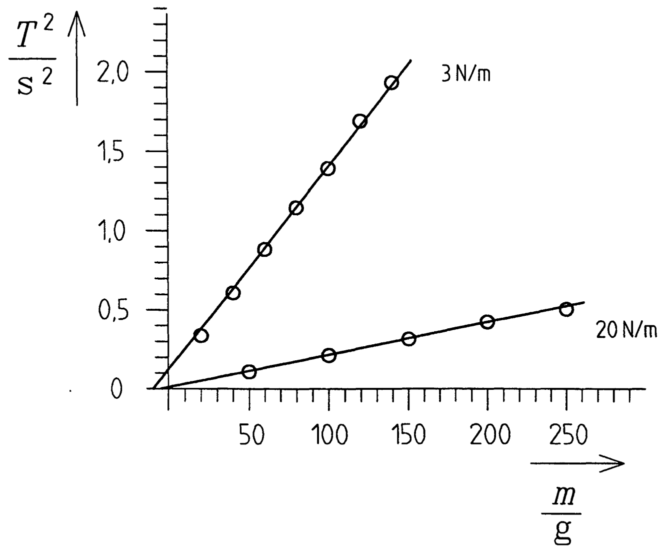
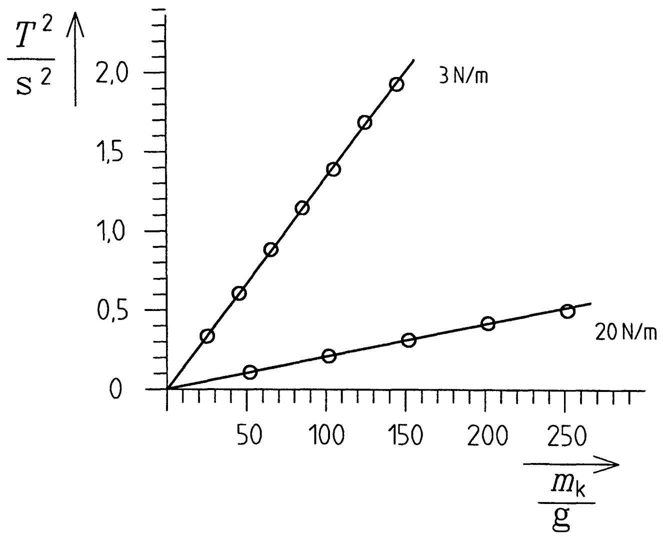


Fig. 3



The m_k values are now calculated in g and in kg; and then recorded in Tables 3 and 4.

Table 3 (20-N/m helical spring, m_F = 5.7 g)

m / g	T^2 / s^2	m_k / g	m_k / kg	$\frac{T^2/m_k}{s^2/kg}$
50	0.109	51.9	0.0519	2.10
100	0.212	101.9	0.1019	2.08
150	0.314	151.9	0.1519	2.07
200	0.422	201.9	0.2019	2.09
250	0.504	251.9	0.2519	2.00

Table 4 (3-N/m helical spring, m_F = 15.8 g)

m / g	T^2 / s^2	m_k / g	m_k / kg	$\frac{T^2/m_k}{s^2/kg}$
20	0.336	25.3	0.0253	13.3
40	0.608	45.3	0.0453	13.4
60	0.884	65.3	0.0653	13.5
80	1.145	85.3	0.0853	13.4
100	1.392	105.3	0.1053	13.2
120	1.690	125.3	0.1253	13.5
140	1.932	145.3	0.1453	13.3

Finally, the quotients T^2/m_k are calculated. These quotients are constants when the measuring accuracy is allowed for and have a mean value of 2.07 s²/kg for the 20-N/m spring and a mean value of 13.4 s²/kg for the 3-N/m spring.
Thus, in both cases the following is true:

$$T^2/m = \text{constant}$$

or

$$T^2 \sim m.$$

This proportional correlation can also be seen in Fig. 3. The straight line for the soft, 3-N/m spring has a much greater slope than the straight line for the hard, 20-N/m spring.
The students are now informed that one can calculate the oscillation period of a spring pendulum with the equation

$$T = 2\pi \cdot \sqrt{m/D}$$

Then

$$T^2/m = 4\pi^2/D$$

or

$$D = 4\,\pi^2/(T^2/m).$$

For the helical spring used first, it follows that:

$$D = 4\,\pi^2/(2.07\ s^2/kg) = 19.1\ kg/s^2 = 19.1\ N/m$$

and for the second one:

$$D = 4\,\pi^2/(13.4\ s^2/kg) = 2.95\ N/m.$$

These values agree well with the values given in the equipment list for the spring constants, which themselves have a tolerance resulting from their manufacture.
In summary, it can be stated that the larger the mass m of the oscillating body (system) and the smaller the spring const $D = f/s$, the larger the oscillation period T of a spring pendulum.
The following is true:

$$T = 2\pi \cdot \sqrt{m/D}$$

and thus

$$T \sim \sqrt{m}$$

as well as

$$T \sim \sqrt{1/D}\,.$$

Remarks

If, for simplification purposes, it is desired to neglect the mass of the springs because the students have difficulty understanding why only one-third of the springs' mass is considered in the calculation, the 3 N/m spring should not be used since the mass error would be considerably greater than that resulting from the measuring inaccuracy.

Space for notes

Physical pendulum

Investigate the properties of physical pendulum.

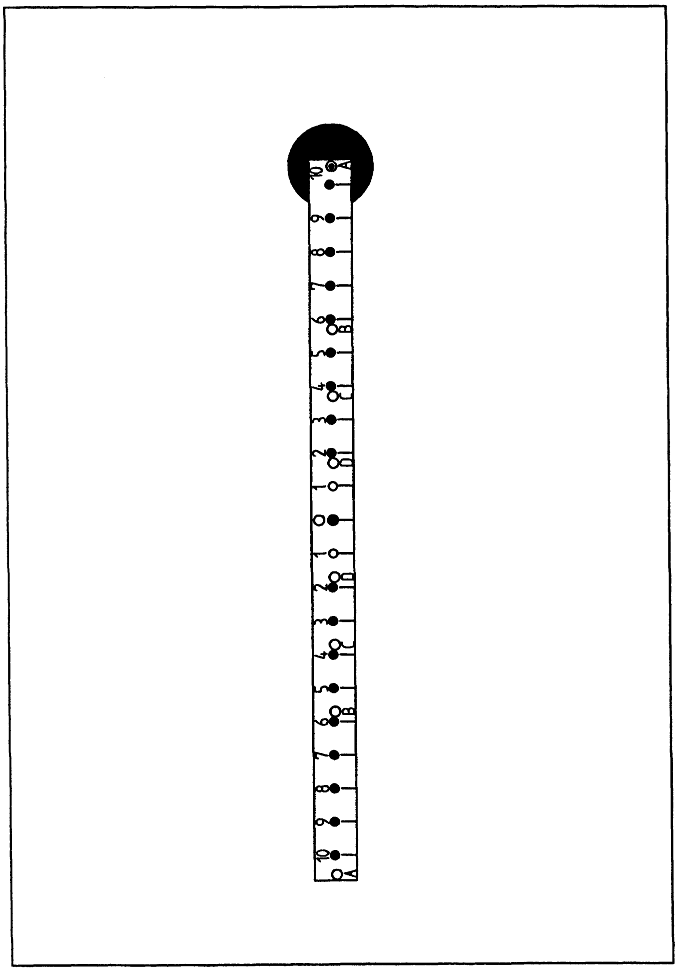
Equipment

Demonstration board for physics	02150.00	1
Clamp on fixing magnet	02151.01	1
Axle on fixing magnet	02151.02	1
Hook on fixing magnet	02151.03	1
Scale for demonstration board	02153.00	1
Weight holder for slotted weights	02204.00	1
Slotted weight, 10 g, silver	02205.02	1
Slotted weight, 50 g, silver	02206.02	1
Holding pin	03949.00	1
Lever	03960.00	1
Fish line, 0.5 m from	02090.00	1
Stopwatch		
White board pen, water soluble		

Set-up and procedure 1

— Place the axle on fixing magnet onto the left half of the demonstration board.

Fig. 1



— Slip hole A on the left side of the lever, which is to be used as a physical pendulum, onto the axle (Fig. 1).

— Allow the lever to oscillate and measure the time which is required for 10 complete oscillations. Record the value for 10 T in Table 1.

— Hang the lever from other points (cf. Table 1) and proceed as described above.

— Measure the distance l_r between the suspension points A on the left and C on the right; record the value for l_r.

Results

Table 1

Suspension point	10 T / s	T / s
A, left	10.8	1.1
B, left	10.2	1.0
C, left	11.0	1.1
D, left	14.4	1.4
C, right	10.8	1.1

l_r = 28.2 cm

Evaluation

The oscillation period of the pendulum has the same value T = 1.1 s in each case, when it oscillates around the points A on the left, C on the left, or C on the right.

The lever which is used as a pendulum has a regular shape. Its mass is uniformly distributed over its length. For this reason, the result $T_{C,left} = T_{C,right}$ could have been expected from the very beginning.

The fact that $T_{A,left} = T_{C,right}$ is also true means that there are two points on the physical pendulum around which the pendulum can oscillate with the same oscillation period. The distance between these two points is termed the reduced pendulum length l_r and for the lever being used as a physical pendulum this distance is l_r = 28.,2 cm.

Accordingly, a thread pendulum with l_r = 28.,2 cm should also have the same oscillation period as the pendulum which can pivot around the suspension points A on the left or C on the right.

The centre of gravity of the physical pendulum (of the lever) lies on the line connecting these two points.

Set-up and procedure 2

— Supplement the set-up for Experiment 1. Place the clamp on fixing magnet onto the right half of the demonstration board. Secure the holding pin in the clamp, and place the hook on fixing magnet to the right of it.
— Thread a piece of cord (fish line) which is approximately 50 cm long through the hole in the holding pin (fig. 2). Attach the cord to the hook and hang the weight holder loaded with a 50-g slotted weight on its other end.
— Adjust the length of the pendulum to 28 cm (28.2 cm) (pendulum length = distance between the pivotal point and the pendulum's centre of gravity, which lies approximately at the top of the 50-g slotted weight).
— Allow the thus created thread pendulum to oscillate. Measure the time 10 T and record it.
— Allow the lever and the thread pendulum to oscillate (the former around point A). Observe them and write down your observation.

Results 2

10 T = 10.9 s, therefore T = 1.1 s

Observation: The two pendulums oscillate synchronously, i.e. they have the same oscillation period.

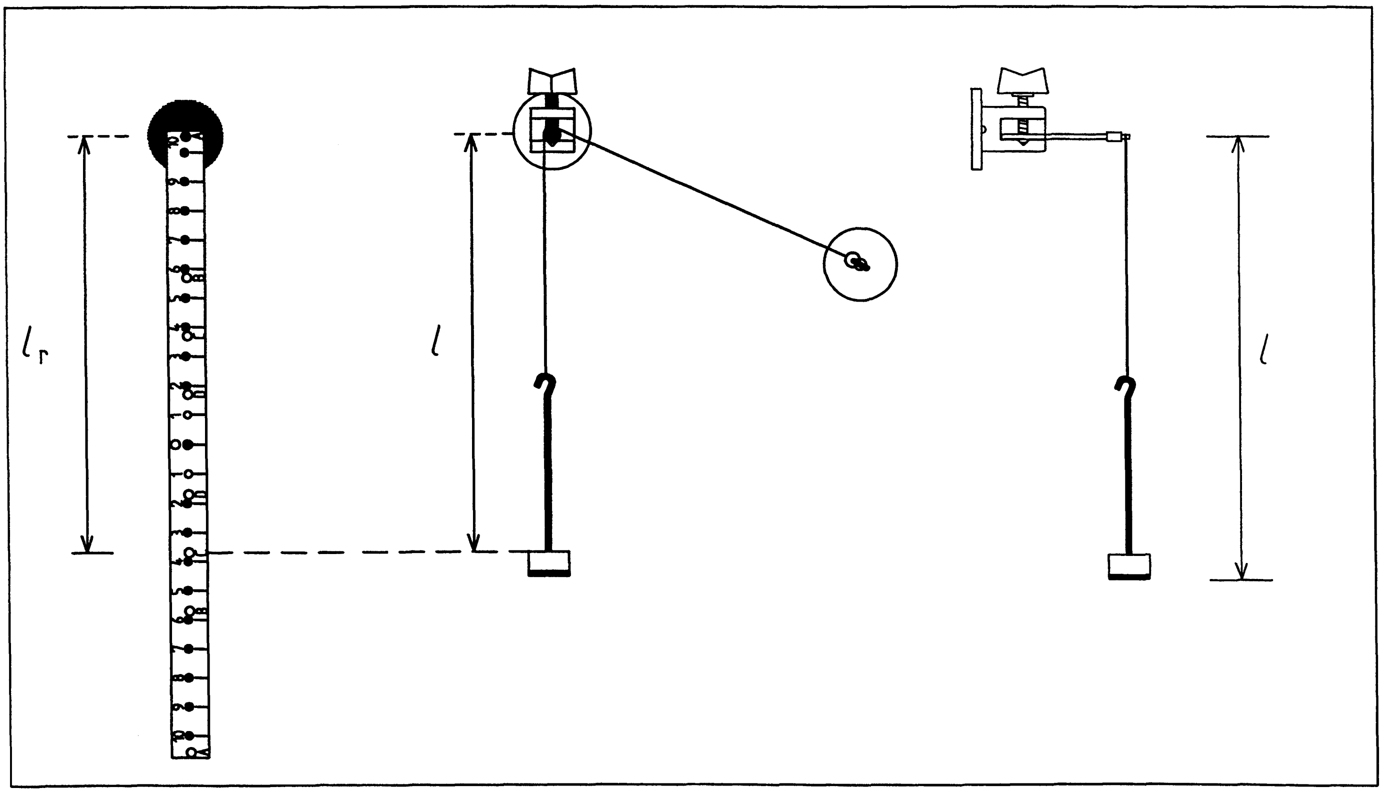
Evaluation 2

The thread pendulum having a pendulum length $l = l_r$ has the same oscillation period as the physical pendulum from Experiment 1, which oscillates around the points A on the left or C on the right.

Label the lengths l and l_r on the demonstration board with the white board pen (as in Fig. 2).

The physical pendulum is also termed a reversion pendulum because it must be reversed if one desires to change to suspension points and still have the pendulum oscillate with the same oscillation period.

Fig. 2



Remarks

The following is true for the oscillation period of the physical pendulum:

$$T = 2\pi \cdot \sqrt{l_r / g} = 2\pi \cdot \sqrt{J_A / (m \cdot g \cdot s)}$$

where

J_A = The moment of inertia for rotation around the axis through the point A

l_r = The reduced pendulum length

m = The mass of the pendulum

s = The distance between point A and the centre of gravity S

In addition, the following is true:

$$J_A = J_S + m \cdot s^2$$

where

J_s = The moment of inertia for rotation around the axis through the centre of gravity S.

For a (thin) rod $J_S = m \cdot l^2/12$

where

l = The total length of the rod (in this case: the lever).

From this it follows that:

$l_r/g = J_A/(m \cdot g \cdot s) = (J_S + m \cdot s^2)/(m \cdot g \cdot s)$,

$l_r/g = (m \cdot l^2/12 + m \cdot s^2)/(m \cdot g \cdot s)$,

$l_r = l^2/(12\,s) + s$.

For the pendulum consisting of the lever where l = 42.8 cm and s = 21 cm:

$l_r = (42.8\text{ cm})^2/(12 \cdot 21\text{ cm}) + 21\text{ cm}$,

$l_r = 7.3\text{ cm} + 21\text{ cm} = 28.3\text{ cm}$ (cf. Evaluation 1).

In Experiment 2 one can also proceed by allowing the physical pendulum to oscillate around point A and by changing the length of the thread pendulum (by moving the hook) until the oscillation periods of the pendulums are equal. If the pendulum length l of the thread pendulum is now measured, one obtains $l = l_r$ as a good approximation.

Space for notes